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Stochastic Herding by Institutional Investment Managers[*†]

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Abstract

This paper demonstrates that the behavior of institutional investors around the downturn of the U.S. equity markets in 2007 is consistent with stochastic herding in attempts to time the market. We consider a model of large number of institutional investment managers who simultaneously decide whether to remain invested in an assets or liquidate their positions. Each fund manager receives imperfect information about the market's ability to supply liquidity and chooses whether or not to sell the security based on her private information as well as the actions of others. Due to feedback effects the equilibrium is stochastic and the "aggregate action" is characterized by a power-law probability distribution with exponential truncation predicting occasional "explosive" sell-out events. We examine highly disaggregated institutional ownership data of publicly traded stocks to find that stochastic herding explains the underlying data generating mechanism. Furthermore, consistent with market-timing considerations, the distribution parameter measuring the degree of herding rises sharply immediately prior the sell-out phase. The sell-out phase is consistent with the transition from subcritical to supercritical phase, whereby the system swings sharply to a new equilibrium. Specifically, exponential truncation vanishes as the distribution of fund manager actions becomes centered around the same action – all sell.

JEL classification codes: D8, G2, G14

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1 Introduction

Many apparent violations of the efficient market hypothesis, such as bubbles, crashes and "fat tails" in the distribution of returns have been attributed to the tendency of investors to herd. Particularly, in a situation where traders may have private information related to the payoff of a financial assets their individual actions may trigger a cascade of similar actions by other traders. While the mechanism of a chain reaction through information revelation can potentially explain a number of stylized facts in finance, such behavior remains notoriously difficult to identify empirically. This is partly because many theoretical underpinnings of herding, such as informational asymmetry, are unobservable and partly because the complex agent-based models of herding do not yield closed-form solutions to be used for direct econometric tests.

This paper attempts to bridge this gap. The 2007 collapse of the U.S. asset bubble has provided researchers with the opportunity to look afresh into the causes of financial instability and crises, including the role played by herding behavior. One of the lesser known chapters in the unravellings of the 2007-2008 crisis has been a substantial sell-off of equities by institutional investors a few quarters before the general market downturn that began in earnest in the summer of 2007. Institutional investors manage between 60 and 70 percent of outstanding U.S. stocks and are regarded as sophisticated investors whose rising importance in capital markets has been extensively documented by Gompers and Metrick (2001) among others. As Figure 1 shows, managers of pension and endowments funds (who account for 48 percent of total market value of S&P 500 stocks or approximately 80 percent of total institutional holdings) began dumping S&P 500 stocks during 2006:Q2 and within four quarters virtually reverted their equity exposure to the pre-2003 level. Forced liquidation cannot explain such marked reduction in institutional stock ownership since at the time major risk indicators were still low and credit markets were not yet under stress.[1] Herding, on the other hand, can provide an alternative explanation. This is because in addition to funding risks institutional investment managers face what Abreu and Brunnermeier (2002, 2003) call "synchronization risk" – the risk of selling an overvalued stock too early, before a critical mass of other investors sells, or too late, after a critical mass of other investors sells. Missing the timing of the price correction in either case would lead to losses and underperformance relative to other managers in the short-run. Such incentive to synchronize with other investment managers due to short time horizons and relative performance considerations (Shleifer and Vishny (1997)) can lead

[1] See Brunnermeier (2009) for the timing of the 2007-2008 liquidity and credit crunch.

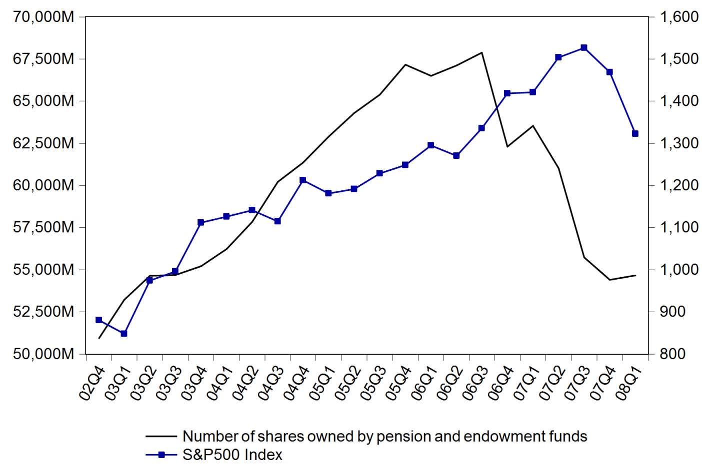

Figure 1: Number of shares (in millions) of S&P 500 stocks held by pension and endowment funds (the largest institutional investor category) and the S&P 500 index.

to herd behavior.

We consider a model of large number of institutional investment managers who simultaneously decide whether to remain invested in an assets or liquidate their positions. The prospect of earning excess returns by riding the trend for an additional time period is weighed against the possibility that a large enough number of fund managers will dump the stock today overwhelming the market liquidity and forcing the price to drop, resulting in losses for those who remain. Each fund manager receives imperfect information about the market's ability to supply liquidity. In Bayesian Nash equilibrium each manager chooses whether or not to continue holding the security based on her private information and the actions of other investment managers.[2] The equilibrium strategy of investment managers exhibits complementarity, since each fund manager is more likely to liquidate when a greater number of others are liquidating. Herding in this environment is stochastic because it turns out that in equilibrium each manager assigns greater weight to the actions of others than her own private information only with a certain probability. In the aggregate, the model predicts a non-trivial probability of "explosive" incidents of uniform coordination on the same action.

Whereas the central limit theorem characterizes an outcome of a simple information aggregation process, choice correlations (e.g. herding) leads to fat tail effects. In particular, the equilibrium

[2]The reliance on the actions of others for information rather than making decision based on prices alone implies that not all interactions between agents are mediated through the market and that these interactions are not anonymous, Cowan and Jonard (2003). For instance, Shiller and Pound (1989) find that word-of-mouth communications are important for the trading decisions of both individuals and institutional investors.

fraction of investment managers that herd on the same action is described by a probability distribution that exhibits exponential decay. This probability distribution can be observed even before the "explosive" sell-out takes place potentially allowing us to quantify what Rajan (2006) has dubbed the "hidden tail risk."[3]

We examine quarterly data from 13F filings with the Securities and Exchange Commission (SEC) in which institutional investment managers report the number of shares under management for each individual security. We find that the distribution of the number of institutional investment managers selling off their shares several quarters before the peak of the S&P 500 index in 2007 is consistent with herding. The parameter capturing the degree of herding behavior rises over time until the first quarter of major institutional sell-off of S&P 500 stocks. The transition to the sell-off itself is consistent with self-organized criticality following Bak et al. (1997). As the exponential decay vanishes in the probability distribution of institutional trades we obtain a (pure) power law distribution. Once that happens, an explosive synchronization occurs sooner or later. Then, through the information revealed by the actions of others, it becomes common knowledge among traders that the bubble has burst. Accordingly, all traders choose sell. However, liquidation needs and other considerations at the fund level imply that traders' behavior may vary due to idiosyncratic reasons. Thus, we only observe an aggregate of idiosyncratic variations in behavior, which leads to a normal distribution due to the Central Limit Theorem. The symmetric behavior is not found on the buy side in line with investors reacting differently to potential losses than to potential gains.

The paper is organized as follows. Section 2 overviews related literature drawing a contrast between herding due to informational cascades and the stochastic herding mechanism employed in this paper. Section 3 presents the model of stochastic herding, derives the equilibrium distribution of herding agents, and conducts numerical simulations of the model. Section 4 examines the distribution of the actions by institutional investment managers from 2003:Q1 through 2008:Q1 covering both the run-up to and the collapse of the most recent U.S. equity bubble. In this section we compare the empirical distribution to the numerical simulations, evaluate the fit of the distribution

[3] Morris and Shin (1999) also argue that choice interdependence among traders must be explicitly incorporated into estimates of "value at risk" and call for greater attention to game-theoretic issues since market outcomes depend on the actions of market participants. One such attempt is made by Nirei and Sushko (2011) who identify key features of foreign exchange speculation that make carry traders susceptible to stochastic herding. The ability of their approach to incorporate "rare" disasters as well as daily volatility in the same data generating process allows to use historical data to quantify the risk a of foreign exchange rate crash even if such an event is not a part of the historical sample. However, unlike the present paper, their approach is less direct as they do not observe the *actions* of traders but rather have to infer their impact from prices.

implied by the model of stochastic herding against several alternatives, and overview the evolution of this behavior over time. Section 5 concludes.

2 Stochastic Herding and Related Literature

Related arbitrage literature includes Shleifer et al. (1990) who show that rational traders will tend to ride the bubble because of risk aversion. Abreu and Brunnermeier (2003) model a continuous time coordination game in which the market finally crashes when a critical mass of arbitrageurs synchronizes their trades. In such a setting, it is futile for well-informed rational arbitrageurs to act on some piece of information unless a mass of other arbitrageurs will do so also. The coordination element coupled with information asymmetries create an incentive for fully rational investors to base their actions on the actions of others, i.e. herd. Scharfstein and Stein (1990), Bikhchandani et al. (1992), Banerjee (1992), and Avery and Zemsky (1998) have formulated a theory of informational cascades, a type of herding that takes place when agents find it optimal to completely ignore their private information and follow the actions of others in a sequential move game.[4] Because players select their actions sequentially the system will eventually but unexpectedly swing from one stable state to another. In contrast, in our framework herding is stochastic following Nirei (2006*b*, 2008) with some foundation going back to probabilistic herding in the famous ant model of Kirman (1993).[5] Only a fraction of agents synchronize, the size of the fraction in turn depends on the realization of private signals. Stochastic herding emerges because strategic complementarity makes it optimal for some agents to place higher value on the informational content of the actions of others' relative to own private signals. This setup differs from pure informational cascades similarly to Gul and Lundholm (1995) in that in our case, as in theirs, none of the information goes unused. As a result of stochastic herding, transition between states only happens with certain probability.

The probability distribution of herding agents is derived from the threshold rule governing their actions. This is similar to the threshold-based switching strategy employed by Morris and Shin (1998) in the global games approach. However, unlike the global games, the threshold value of the signal determining whether or not an the investment manager chooses to liquidate her position fluctuates endogenously with the actions of others. Endogenously fluctuating threshold can

[4] See Chari and Kehoe (2004) for the application of information cascades to financial markets.

[5] Alfarano et al. (2005) and Alfarano and Lux (2007) extend the Kirman model in a different direction: they focus on the ability of the model with asymmetric transition probabilities of different types of traders to match higher moments in financial returns, whereas the stochastic herding approach focuses on the mapping of heterogeneous information onto the agents' action space.

generate cascading behavior whereby agents continuously lower their threshold belief for liquidating an assets as they observe more and more liquidation around them. This leads to a non-trivial possibility of an "explosive" event in which the vast majority of investment managers liquidate simultaneously causing the liquidity to dry up. In this manner, we show that even if private signals about future market liquidity are normally distributed, the resulting aggregate action will follow a highly non-normal distribution implying stylized facts such as volatility clustering and fat tails in the distribution of financial returns.[6]

Finally, agents are rational but myopic. This feature is particularly suitable for modeling fund manager behavior whose performance is often evaluated on short-term basis and relative to other managers. Our model is intended to explain fund manager choice of action at quarterly frequency so implicitly we assume that each manager optimizes with one quarter ahead horizon. Another class of investors whose behavior we do not model include individual investors and managers of funds with substantial restrictions on customer redemptions, access to a wider variety of investment instruments, and subject to less stringent regulations. These investors operate at a different performance horizon and have served as liquidity providers during such episodes as the 1987 stock market crash (Fung and Hsieh (2000)) to the more constrained institutional investors such as pension funds, endowment funds, and insurance companies that we focus on in this study.

Empirical studies of herding have mostly focused on abnormal changes in institutional portfolio composition as evidence of herding (see Nofsinger and Sias (1999), Kim and Nofsinger (2005), and Jeon and Moffett (2010) for the ownership change portfolio approach).[7][8] Sias (2004) examines herding among institutional investors in NYSE and NASDAQ by using a more direct measure that looks at the correlation in the changes of an institution's holdings of a security with last period changes in holdings of other institutions. Our empirical approach is more closely related to Alfarano et al. (2005) and Alfarano and Lux (2007), in that we examine the goodness of fit of the empirical distribution to the theoretical distribution implied by the model instead of performing quantile or regression analysis like the earlier works.

We utilize two additional sources of variation in stock holdings not commonly found in data: the variation across individual investors and the variation across a group of closely related securities.

[6] Our approach also bears some relationship to the studies of markets for information such as Veldkamp (2006) who identifies herding as an element of intrinsic instability because it makes markets respond disproportionally to seemingly trivial news.

[7] In related empirical studies McNichols and Trueman (1994) finds herding on earnings forecasts, Welch (2000) finds that security analysts herd, and Li and Yung (2004) finds evidence of institutional herding in the ADR market.

[8] Laboratory studies of herding in speculative attacks include Brunnermeier and Morgan (2004) and Cheung and Friedman (2009).

This means that instead of observing one realization of the aggregate action during each period one can observe a sample of data points large enough to get an insight into the underlying data generating mechanism by looking at its distribution. Each observation in the sample is a group of institutional investors that fall within same class (e.g. banks, pension funds, etc.) holding the same stock. If investors are unsure about the accuracy of their private signal about future market liquidity and are prone to follow the actions of others within the same stock-investor-type group, then, because of the complementarity of their market-timing strategies, the probability of observing large outliers is much higher compared to the case when investors act independently. Specifically, if the behavior of institutional investment managers can be described by stochastic herding then the distribution of their actions will exhibit exponential rather than Gaussian decay. Moreover, the exponential decay will vanish and the distribution will approach a pure power law in the state of self-organized criticality when any large size of herding can occur with a considerable probability.

In a related work, Gabaix (2008) describes a number of data generating processes with feedback effects that have been known to produce power law distributions. However, we depart from their approach in several ways. Gabaix et al. (2006) derive power-law scaling in trading activity from the power-law distribution in the size of the traders, while we obtain this result from the interactions of same-size traders. In other words, we obtain power-law scaling without imposing parametric assumptions on exogenous variables. Instead, it suffices that the signals about the true state are informative in the sense of satisfying the Monotone Likelihood Ratio Property (MLRP). For instance, as in this paper, the information and the true state can follow a bivariate normal distribution. One advantage of developing this empirical approach is its potential, given the right data, to quantify the "hidden tail risk" and provide advance warning of an impeding instability by identifying a system with high degree of choice interdependence based on the distribution of aggregate action.

3 Model

3.1 Threshold Switching Strategy

In this section, we present a model of stochastic herding of informed traders. Our model setup is motivated by Abreu and Brunnermeier (2003) in which traders try to time their exit from a bubble maket. In this setup, we apply an analytical tool shown by Nirei (2006*b*) in order to obtain the distributional pattern of traders' herding. This distributional form then motivates our empirical investigation in the next section on the distributions of the herd size of institutional traders before and during the sell-out period.

There are N informed institutional investment managers indexed by $i = 1, 2, \dots, N$, for conciseness we will refer to them simply as traders. Each trader is endowed with one unit of risky asset. The trader gains $(g - r)p$ by riding on bubbles and loses βp if the bubble bursts. Trader i can either sell ($a_i = 1$) or remain in the same position ($a_i = 0$). Each trader observes the aggregate number of selling traders $a \equiv \sum_{i=1}^{N} a_i$ and a private signal x_i. Let α denote the fraction of selling traders $\alpha = a/N$.

Market liquidity is denoted by θ.[9] The informed traders cannot observe θ, but only observe a noise-ridden proxy $x_i = \theta + \epsilon_i$. x_i is a private information and ϵ_i is independent across traders.

The bubble bursts if the selling pressure by the informed traders overwhelms the liquidity provided by the noise traders. The burst occurs if $\alpha > \theta$. Informed traders' expected utility of holding the asset is:

$$(g - r)p \Pr(\theta \geq \alpha \mid x_i, a, a_i = 0) - \beta p \Pr(\theta < \alpha \mid x_i, a, a_i = 0), \tag{1}$$

and the expected utility of selling is 0. Then the optimal strategy is to sell if:

$$\frac{g - r}{\beta} < \frac{\Pr(\theta < \alpha \mid x_i, a, a_i = 0)}{\Pr(\theta \geq \alpha \mid x_i, a, a_i = 0)}, \tag{2}$$

or, equivalently,

$$\frac{g - r}{\beta} < \frac{\Pr(x_i, a, a_i = 0, \theta < \alpha)}{\Pr(x_i, a, a_i = 0, \theta \geq \alpha)}, \tag{3}$$

and hold otherwise.

3.2 Equilibrium

We make a guess that all traders follow a threshold rule that trader i sells if $x_i \leq \bar{x}(\alpha)$ and holds otherwise. We will verify this guess later. We consider a Nash equilibrium in which each trader does not have an incentive to deviate from the threshold rule at any observation (x_i, α), given that all the other traders obey the rule. When there are multiple equilibria for a realization of the private information (x_i), the outcome with the smallest a is selected. We denote the selected equilibrium

[9] θ represent the liquidity provided by market participants other than institutional investors, such as individual investors or alternative investors (e.g. hedge funds) with lock up periods and greater choices of investment instruments thus not subject to short-term performance considerations. We do not model their behavior, hence refer to this group simply as noise traders.

by a^*. This equilibrium can be implemented by submission of supply schedule to a market maker. In this scheme, each trader submits their action of selling or holding conditional on α, and the market maker selects the smallest α such that it is equal to the aggregate supply conditional on α. The equilibrium can be interpreted as the outcome of a sequential trading where informed traders can sell immediately after observing the selling of other traders.

Define:

$$G(\bar{x}, a) = \Pr(x_j > \bar{x}(a) \mid \theta < a/N) \tag{4}$$

$$F(\bar{x}, a) = \Pr(x_j > \bar{x}(a) \mid \theta \geq a/N) \tag{5}$$

$$A(\bar{x}, a) = G(\bar{x}, a)/F(\bar{x}, a) \tag{6}$$

$$\delta(x_i, a) = \Pr(x_i, \theta < \alpha)/\Pr(x_i, \theta \geq \alpha) \tag{7}$$

$A(\bar{x}, a)$ represents the information revealed by a holding trader at the observed supply a. The information is expressed in the form of an odds ratio. $\delta(x_i, a)$ is the odds ratio obtained by the private information x_i.

Under the guessed threshold policy, the joint probability in (2) can be decomposed by the information revealed by the actions of traders. For example, when $a = 0$, the joint probability is written as:

$$\Pr(x_i, a = 0, a_i = 0, \theta < 0) = \Pr(x_i \mid \theta < 0) \Pr(x_j > \bar{x}(0) \mid \theta < 0)^{N-1} \Pr(\theta < 0) \tag{8}$$

Then, (3) is rewritten for $a = 0$ as:

$$\frac{g - r}{\beta} < A(\bar{x}(0), 0)^{N-1} \delta(x_i, 0) \tag{9}$$

Thus, $\bar{x}(0)$ is implicitly determined by:

$$\frac{g - r}{\beta} = A(\bar{x}(0), 0)^{N-1} \delta(\bar{x}(0), 0) \tag{10}$$

Now consider the case $a > 0$. If $a > 0$ were chosen to be an equilibrium, it reveals that no smaller supply $a' = 0, 1, \ldots, a - 1$ is consistent with the supply schedule, since the market maker chooses the smallest α that is consistent with the supply schedule. Thus, the equilibrium reveals not only that there are a traders who sell conditional on a, but also that there are at least $a' + 1$

traders who sell at a' for each $a' < a$.

Therefore, there are a traders with private information $x_i < \bar{x}(a)$, there are at least a traders with private information $x_i < \bar{x}(a-1)$, there are at least $a-1$ traders with $x_i < \bar{x}(a-2)$, and so forth up to that there is at least 1 trader with $x_i < \bar{x}(0)$. This set of conditions is equivalent to that there is one trader in each region $x_i < \bar{x}(a')$ for all $a' = 0, 1, \ldots, a-1$.

Consider the trader who would hold at $a'-1$ but sell at a'. Define the information revealed by such a trader at equilibrium a as follows:

$$B(\bar{x}(a'), a) = \frac{\Pr(x_j \leq \bar{x}(a') \mid \theta < a/N)}{\Pr(x_j \leq \bar{x}(a') \mid \theta \geq a/N)}. \tag{11}$$

Then, the selling condition (3) is rewritten for $a = 1$ as:

$$\frac{g-r}{\beta} < \delta(x_i, 1) A(\bar{x}(1), 1)^{N-2} B(\bar{x}(0), 1). \tag{12}$$

Then, $\bar{x}(1)$ is determined by $x_i = \bar{x}(1)$ that equates the both sides above. Generally, the threshold $\bar{x}$ is determined recursively by the equation:

$$\frac{g-r}{\beta} = \delta(\bar{x}(a), a) A(\bar{x}(a), a)^{N-1-a} \prod_{k=0}^{a-1} B(\bar{x}(k), a) \tag{13}$$

for $a = 0, 1, 2, \ldots, N-1$. We note that the posterior likelihood in (3) has three components: the private information x_i, the information revealed by holding actions of $N-1-a$ traders, and the information revealed by selling actions of a traders.

We assume that the prior belief on θ and the noise ϵ_i jointly follow a bivariate normal distribution with mean $(\theta_0, 0)$ and variance (σ_0^2, σ_e^2). Then (θ, x_i) also follows a bivariate normal distribution, since $x_i = \theta + \epsilon_i$. The normal distribution implies that:

$$\Pr(x_j > \bar{x} \mid \theta) < \Pr(x_j > \bar{x} \mid \theta'), \quad \text{for any } \theta < \theta'. \tag{14}$$

Thus,

$$A(\bar{x}, a) = \frac{\Pr(x_j > \bar{x} \mid \theta < a/N)}{\Pr(x_j > \bar{x} \mid \theta \geq a/N)} < 1 \tag{15}$$

for any a and $\bar{x}$. Likewise,

$$B(\bar{x}, a) = \frac{\Pr(x_j \leq \bar{x} \mid \theta < a/N)}{\Pr(x_j \leq \bar{x} \mid \theta \geq a/N)} > 1. \tag{16}$$

The threshold policy has the following property.

Proposition 1. *The threshold function $\bar{x}(a)$ is increasing in a.*

Proof: See Appendix A.

Using the increasing threshold strategy, we obtain the existence of an equilibrium. To see that, define an aggregate response function as $\Gamma : \{0, 1, \ldots, N\} \mapsto \{0, 1, \ldots, N\}$ for a fixed realization of (x_i). Γ maps the observed a to the number of traders who decide to sell upon the observation a', given (x_i). Then, a' is the number of traders with $x_i < \bar{x}(a)$. Since $\bar{x}$ is increasing in a, Γ is a non-decreasing step function. Hence Γ has a fixed point in $\{0, 1, \ldots, N\}$ by Tarski's fixed point theorem.

Proposition 2. *An equilibrium a^* exists for each realization of a vector (x_i).*

Proof: See Appendix B.

Next, we construct a fictitios tatonnement process that converges to the equilibrium a^* as a means to characterize the equilibrium. First, we define $-H'/H$ as the hazard rate for the traders who have remained holding the asset to sell upon observing a. Let θ_1 denote the true parameter for the liquidity θ. Then $H(\bar{x}) = \int_{\bar{x}} e^{-\frac{(x_j - \theta_1)^2}{2\sigma_e^2}} dx_j / \sqrt{2\pi}\sigma_e$. We define $\mu(a)$ as the mean number of traders who do not sell upon observing $a - 1$ but decide to sell upon observing a. Then:

$$\mu(a) = (H(\bar{x}(a-1)) - H(\bar{x}(a)))(N - a). \tag{17}$$

$\mu(a)$ is also expressed by the product of the increment in the threshold $\bar{x}(a+1) - \bar{x}(a)$, the hazard rate, and the number of traders who continue to hold the asset:

$$\mu(a) \sim \frac{H'}{H}(\bar{x}(a+1) - \bar{x}(a))(N - a) \to \frac{H'/H}{F_1/F} \frac{\log(B/A) + (\partial A/\partial \alpha)/A}{(G_1/F_1)/A - 1} \tag{18}$$

Now, as a fictitious tatonnement process, we consider a best response dynamics $a_{u+1} = \Gamma(a_u)$ that starts from $a_0 = 0$, where a_{u+1} denotes the number of traders with private information $x_i < \bar{x}(a_u)$. We can show that the best response dynamics can be regarded as a tatonnement which converges to the selected equilibrium a^*.

Proposition 3. *For any realization of θ and (x_i), the best response dynamics a_u converges to the equilibrium selected by the market maker, a^*.*

Proof. Suppose that the best response dynamics did not stop at a^*. Then there exists a step s so that $a_s < a^* < a_{s+1}$. But, the definition of a^* prohibits that there is any $a' < a^*$ such that the number of traders with $x_i < \bar{x}(a')$ exceeds a^*. Hence, there is no such s. □

Unconditional on the realization of the private information, (a_u) can be regarded as a stochastic process. In the first step, a_1 follows a binomial distribution with population N and probability $\bar{x}(0)$. In the subsequent steps, the increment $a_{u+1} - a_u$ conditional on a_u follows a binomial distribution with population $N - a_u$ and probability $H(\bar{x}(a_{u-1})) - H(\bar{x}(a_u))$.

As $N \to \infty$, the binomial distribution asymptotically follows a Poisson distribution with mean $(N - a_u)(H(\bar{x}(a_{u-1})) - H(\bar{x}(a_u)))$. Now consider a special case where $\mu(a)$ defined in (17) is constant across a asymptotically as $N \to \infty$. Then, the asymptotic mean of the Poisson distribution above becomes $(a_u - a_{u-1})\mu$. A Poisson distribution with this mean is equivalent to $(a_u - a_{u-1})$-times convolution of a Poisson distribution with mean μ. Thus, in this particular case, the best response dynamics asymptotically follows a branching process with a Poisson distribution with mean μ, which is a population process that starts with the "founder" population with a_1 and each "parent" bears "children" whose number follow the Poisson with mean μ. The selected equilibrium a^* is the cumulated sum of the branching process. The following is known for the distribution function of the cumulated sum of a branching process.

Theorem 1. *Consider a branching process b_u, $u = 1, 2, \ldots$, in which the number of children born by a parent is a random variable with mean μ.*

1. *When $b_1 = 1$, the cumulated sum $Z = \sum_{u=1}^{\infty} b_u$ follows:*

$$\Pr(Z = z \mid b_1 = 1, z < \infty) \sim c^{-z} z^{-1.5} \tag{19}$$

   for large z and for a constant $c \geq 1$ with the equality holding if and only if $\mu = 1$.

2. *The branching process converges to zero in a finite time u with probability one if and only if $\mu \leq 1$.*

3. *If $\mu > 1$, The cumulated sum Z is infinite with a non-zero probability.*

4. *If the number of children born by a parent follows a Poisson distribution with mean μ, then:*

$$\Pr(Z = z \mid b_1) = (b_1/z) e^{-\mu z} (\mu z)^{z - b_1} / (z - b_1)! \tag{20}$$

for $z = b_1, b_1 + 1, \ldots$.

5. *In addition to the previous assumption, if* b_1 *follows a Poisson distribution with mean* μ_1*, then:*

$$\begin{aligned} \Pr(Z = z) &= \mu_1 e^{-(\mu z + \mu_1)} (\mu z + \mu_1)^{z-1} / z! &(21)\\ &\sim (\mu e^{1-\mu})^z z^{-1.5} &(22) \end{aligned}$$

The first item in this theorem implies that the number of traders in a herd follows a non-normal distribution function which exhibits a power-law decay with exponential truncation. Moreover, the distribution of the herd size exhibits a pure power law when $\mu = 1$, since then the exponential term c disappears. The second item implies in our model that the best response dynamics converges with probability one if $\mu < 1$, and thus it verifies that the best response dynamics serves as a valid fictitious tatonnement in this case. The third item implies that there is a positive probability for an "explosive" event if $\mu > 1$. In our model, this event corresponds to the equilibrium in which all traders sell. The fourth and fifth items further characterize the herd size distribution. This particular distribution forms our preferred hypothesis in the empirical investigation of the herd size distribution in the next section.

3.3 Numerical Simulations

Before we move on to our empirical investigation, we numerically compute the model threshold $\bar{x}(a)$ and the equilibrium α^*. The purpose of this simulation is to show that the herd size distribution of the exact equilibrium α^* follows the same distribution as taht we obtained above analytically under approximation. We set the parameter values as follows. The number of institutional investors is $N = 160$. The return from riding the bubble is $g = 0.1$, the interest rate is $r = 0.04$, and the discount by the burst of the bubble is $\beta = 0.82$. The liquidity θ follows a normal distribution with mean 0.5 and standard deviation 0.3. The noise ϵ_i follows a normal distribution with zero mean and standard deviation 1.

Figure 2 plots the threshold function $\bar{x}(a)$ and the conditional mean function $\mu(a)$. The plot is truncated at the point $a = 140$, since for higher a we could not compute $\bar{x}$ because it is too large.

We then simulate the distribution of equilibrium a. We compute a for each draw of a random vector (ϵ_i), and iterate this for $100,000$ times. We observe $a = 0$ for $72,908$ times, and observe

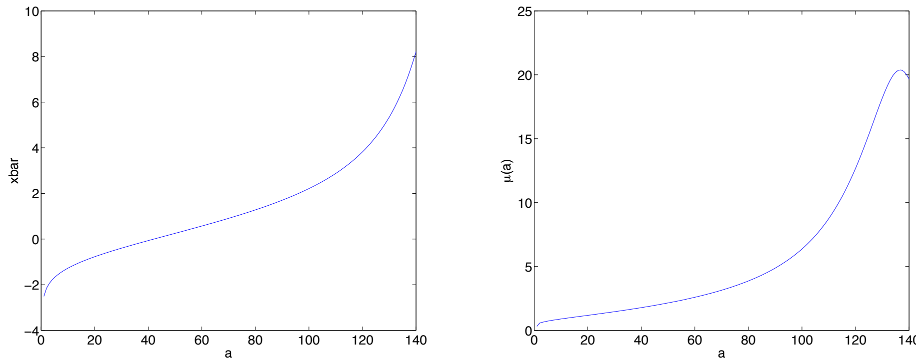

Figure 2: Left: threshold function $\bar{x}(a)$; Right: conditional mean $\mu(a)$

$a = 140$ (the upper bound) for 1215 times. Figure 3 plots the histogram of the all 100,000 observations. In Figure 3, it is clear that a is distributed similarly to an exponential distribution for $0 < a < 50$. There is no incident of $a > 50$ except for the 691 "explosive" incidents in which case basically all the traders decide to sell.

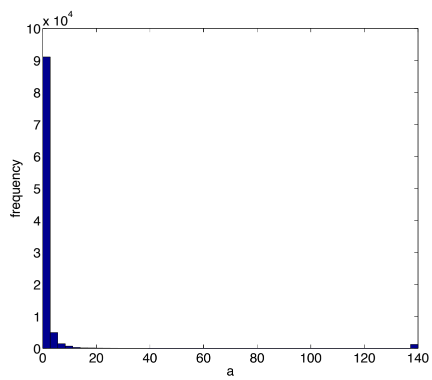

Figure 3: Histogram of a for $0 \leq a \leq 140$

Figure 4 plots the blowups of the histogram for $0 < a < 160$. The left panel plots the histogram in linear scale and the right panel plots it in semi-log scale. The tail distribution exhibits a straight line on semi-log scale that is characteristic of exponential decay. This is due to the persistent outliers that arise from propagation effects in the underlying data generating process. The shape of the probability density function of the equilibrium distribution of a derived in the model (21) is illustrated in Figure 9 in Appendix C.

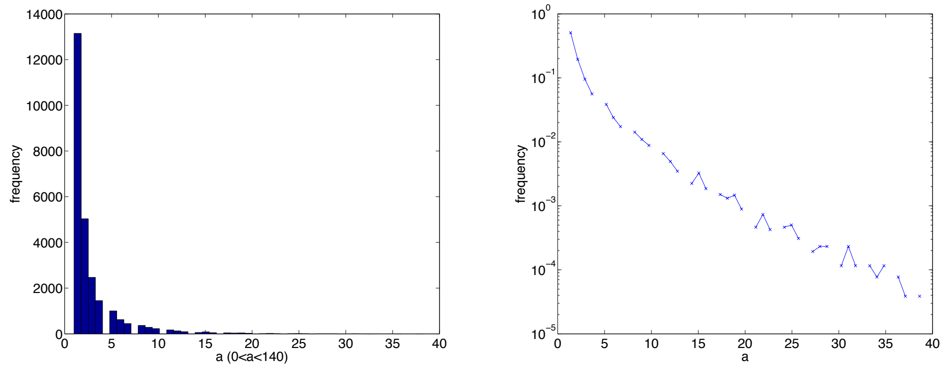

Figure 4: Histogram of a for $0 < a < 140$

4 Evidence from Institutional Equity Holdings

4.1 Data Description and the Unit of Observation

We study the behavior of institutional investment managers around the latest run-up and the subsequent collapse of the U.S.stock market associated with the asset bubble of the 2000s. Specifically, we examine institutional investor holdings of stocks included in the S&P 500 index during the period from 2003:Q1 through 2008:Q1. As has been discussed in the introduction, institutional investors increased their equity holdings markedly between 2003:Q1 and 2006:Q1 after which point the majority of them, especially pension and endowment funds, began reducing their stock portfolios to the pre 2003 levels. This episode provides a unique opportunity to examine the role played by herding in the propagation of such massive adjustment. Herding behavior is especially suspect because this marked adjustment in institutional portfolios preceded the onset of the credit crisis and cannot be attributed to forced liquidations.

We use data on institutional equity holdings from Spectrum database available through Thompson Financial.[10] The data is complied from quarterly 13F filings with SEC in which institutional investment managers with over $100 million under discretionary management are required to report their long positions in exchange traded stocks, closed-end investment companies, equity options and warrants.

Table 1 shows the breakdown of institutional investment managers in our sample by type for each quarter from 2003:Q1 through 2008:Q1. Pension and endowment funds comprise the largest re-

[10]Studies that utilize 13F data include Gompers and Metrick (2001), Brunnermeier and Nagel (2004), Sias (2004), and Hardouvelis and Stamatiou (2009)

porting category ranging between 71% and 80% of all instutional investment managers. Investment advisers comprise the second largest categoy followed by investment companies, insurance companies, and banks. For instance, in 2008:Q1 the dataset includes 2,119 pension and endowmnets funds, 521 investment advisers, 96 investment companies, 19 insurance companies, and 9 banks.

As Table 2 illustrates, institutional investors hold the majority of outstanding U.S. equities, as proxied by the S&P 500 stocks. The share of institutional holdings rose from 53% in 2003:Q1 to 67% in 2006:Q1 then declined steadily through 2008:Q1. Pension and endowment funds are the most dominant category accountring for more than four fifth of total institutional holdings of S&P 500 stocks.

The high degree of disaggregation in the Spectrum data allows us to group institutional investment managers into stock-investor-type groups, $N(j, k)$, where j indicates an S&P500 stock and k indicates institutional investor type. For example, N(APPL, Banks and Trusts) is the number of banks and trusts that own Apple stock. Only groups with 10 traders or more are included in the sample. Table 3 shows the summary statistics. The number of quarterly observations for $N(j, k)$ ranges from 1,535 to 1,882. The size of the groups varies considerably, with quarterly mean ranging from 114 to 146, and quarterly maximum ranging from 1,046 and 1,222. Each quarter $a(j, k)$ out of $N(j, k)$ institutions in each group liquidate their holdings. Institutional managers dumping more than 80% of their holdings are counted into $a(j, k)$, but the results are generally robust to different cutoff levels.[11]

4.2 Summary Statistics

Table 4 shows quarterly summary statistics for $a(j, k)$. Note the stark difference between 2006:Q2 through 2007:Q1 and the surrounding quarters. During 2006:Q2 through 2007:Q1 the mean $a(j, k)$ is between 104 and 117 compared to 2 and 4 in other quarters and the maximum during this four quarter period ranges from 1057 to 1114 compared to 23 and 347 during other quarters. The corresponding *fraction* of institutions liquidating a stock, $a(j, k)/N(j, k)$, controls for any group size effect in the values of $a(j, k)$. Table 5 shows the summary statistics for $a(j, k)/N(j, k)$ confirming that during the period of 2006:Q2 through 2007:Q1 is associated with large a large

[11] The model of stochastic herding yields prediction regarding an "extreme" event, namely a complete liquidation of a position in a security. Realisitically, large block holders, such as institutional investors, are restricted in their ability to unload a substantial number of shares at once, therefore we interpret the sale of 80% or greater share as an extreme event. The results are robust to different levels of cutoff, however, choosing the cutoff at 100% as stipulated by the model greatly reduces the number of observations while missing valuable information contained in extremely large sales approaching 100%.

liquidation of stocks by institutional investment managers. The mean fraction of institutional managers liquidating a stock jumped to the 79% and 89% range from the earlier range of 3% to 4%. Moreoever, during this four quarter period some stock-investor type groups experienced complete liquidation as seen from the maximum $\alpha(j,k)$'s of 100%. In sum, the summary statistics of $a(j,k)$ and $a(j,k)/N(j,k)$ in Tables 4 and 5 illustrate a regime change in institutional equity holdings during 2006:Q2 through 2007:Q1 when the vast majority were dumping their S&P 500 stocks. We refer to this period as the sell-out phase.

Focusing on the two quarters immediately preceding the sell-out phase, the summary statistics of $a(j,k)$ and $a(j,k)/N(j,k)$ show a rise in both mean and maximum values compared to previous quarters indicating a possible shift in institutional investment managers' behavior beginning to take place. The mean of $a(j,k)$ increased to 4 during 2005:4 and 2006:Q1 compared to 2 to 3 during all preceding quarters (Table 4) and the maximum $a(j,k)$ is 105, more than double the value during the four preceding quarters. The corresponding fraction, $\alpha(j,k)$, also rose during 2005:Q4 and 2006:Q1 compared to the preceding quarters (Table 5). This increase in the average and in the tail of the distribution of aggregate selling behavior may indicate greater degree of synchronization immediately before the regime change in 2006:Q2. In the remainder of the section we conduct distributional analysis motivated by the model of stochastic herding to examine whether the fat tail in the distribution of $a(j,k)$ during the run-up to the sell-out phase is a result of greater choice correlations and herding by institutional investment managers as opposed to being driven by uncorrelated events.

4.3 Analysis of distribution

The left panel of Figure 5 shows the histogram empirical $a(j,k)$ for the entire sample period (2003:Q1 through 2008:Q1). The histogram bears close similarity to the numerical simulations of the model in Figure 3. Like the simulated a, the distribution of empirical $a(j,k)$ exhibits exponential decay in the high probability mass region of low number of sellers along with a long tail indicating high probability of large outliers. The mean number of institutional fund managers dumping a particular stock is 23, while standard deviation is 79 and the maximum is 1114.

To control for rare events on the "buy side" we also examine a symmetric indicator to $a(j,k)$ for fund managers who increase their holdings of an S&P 500 stock by more than 5 times (inverse of 0.80) during a given quarter, $b(j,k)$. For each stock-investor-type group we then construct the net measure as $a(j,k) - b(j,k)$ and normalize it by group size $N(j,k)$. The right panel of Figure

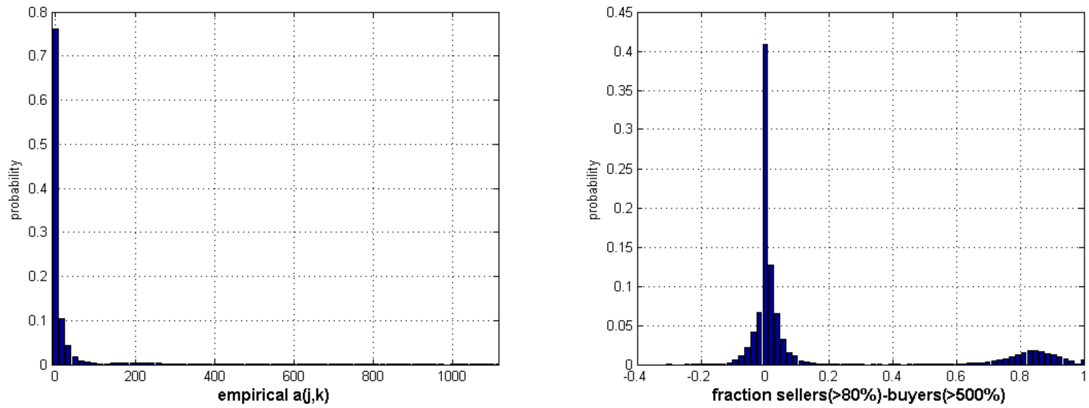

Figure 5: 2003:Q1 - 2008:Q1 (38,353 observations); *Left* histogram of empirical $a(j,k)$. *Right* histogram of $(a(j,k) - b(j,k))/N(j,k)$.

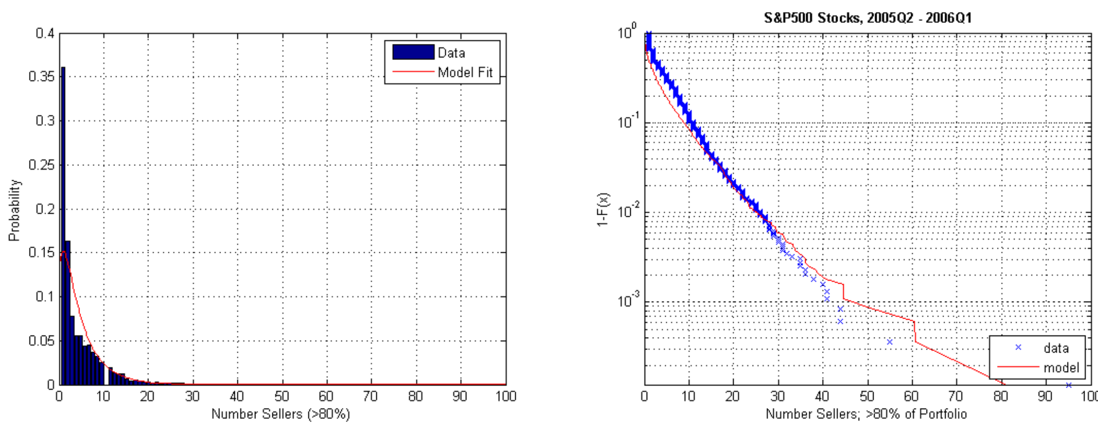

Figure 6: 2005:Q2 - 2006:Q1: *Left* histogram of empirical $a(j,k)$. *Right* semi-log probability plot of empirical $a(j,k)$ and the fitted model (red).

5 shows the histogram of the corresponding fraction. The bimodality of the distribution indicates the presence of "explosive" sellout events, with virtually no observations in the intermediate range. Moreover, such extreme switching from low activity to high activity level is only present on the sell side, indicating that coordination on the same action characterizes sellouts but not purchases by institutional fund managers.

Independent rare events, such as portfolio liquidations due to idiosyncratic shocks, should be well approximated by a Poisson distribution. On the other hand, chain reaction through information revelation will cause $a(j,k)$ to be distributed according to Equation (22) (*Theorem 1*). Recall that in Equation (22), μ_1 represents the Poisson mean of the number of agents taking extreme action at the beginning of the tatonnement process independently (responding only to their private signal), while μ represents the total number of agents induced to follow the actions of the first-mover until the system settles at a new Bayesian Nash equilibrium. In other words, μ quantifies the degree of herding. If $\mu = 0$ then Equation 22 reduces to a probability density function of a Poisson distribution with arrival rate μ_1 indicating the absence of herding (portfolio liquidations are independent of each other). On the other hand, as $\mu \to 1$ the system tends to self-organized criticality with "explosive" convergence on the same action. In the intermediate range, the probability distribution of $a(j, k)$ will exhibit exponential decay, with the speed of the decay dictated by μ. We can also think of μ as a measure of length of the tail of the distribution – larger μ implies that an initial outlier (itself governed by Poisson arrival rate μ_1) attracts greater probability mass to itself, effectively stretching the tail.

The common benchmark distribution for rare independent events is Poisson. Table 6 shows the results of Kolmogorov-Smirnov goodness of fit test for Poisson distribution to $a(j, k)$. Poisson distribution is rejected at the 5% significance level with p-value=0 and the test statistic of 0.769 (three orders of magnitude larger than the critical value of 0.008). Apart from correlated arrivals, Poisson may also be rejected because the distribution of a discrete random variable with Poisson arrival rate asymptotes to normal in the limit. However, as Table 7 shows, the moments of $a(j, k)$ point at a highly non-normal distribution (consistent with the histograms in Figure 5). If the correlated arrival results from stochastic herding then Equation (22) should adequately characterize the probability distribution of emprical $a(j, k)$. Table 8 shows the associated maximum likelihood estimates (MLE) of the distribution parameters. The estimates for μ_1 and μ are 2.058 and 0.938 and are statistically significant at 1% level, indicating that stochastic herding is a plausible candidate for the underlying data generating mechanism of empirical $a(j, k)$.

Figure 6 focuses on the four quarter period before the sell-out phase (2005:Q2 through 2006:Q1). The left panel of Figure 6 shows the histogram of empirical $a(j, k)$ with distribution exhibiting exponential tail similar to the simulation in Figure 4. The largest value in the histogram corresponds to 95. The right panel of Figure 6 shows the corresponding semi-log probability plot. The straight line formed by the observations of $a(j, k)$ on the semi-log scale indicates an exponential distribution with persistent outliers, indicative of correlated arrivals in the underlying data generating process. The solid line shows the fit corresponding to the stochastic herding outcome (of Equation 22) to the empirical distribution of $a(j, k)$. The line was formed by sampling the data from the proportional theoretical probability density (Equation 22) with parameters first estimating using empirical $a(j, k)$

via MLE and the proportionality constant set equal to the theoretical prediction for the power exponent of 1.5.

4.4 The Sell-Out Phase in 2006:Q2-2007:Q1

Figure 7 plots $a(j,k)/N(j,k)$ against the cumulative distribution (log rank over the number of observations). The left panel corresponds to the 2005:Q2 through 2006:Q1 period, the four quarters preceding major institutional sales. The inverse of the slope of the semi-log plot provides an estimate of the mean parameter of an exponential distribution. A least squares regression for $a(j,k)/N(j,k)$ yields an estimate of the slope of -31.443 (standard error 0.055) with an R-squared 0.988. This examination of the semi-log plots favors a model that generates exponential rather than normal decay in $a(j,k)/N(j,k)$ during the final phase in the run-up to the shift in institutional behavior in 2006:Q2.

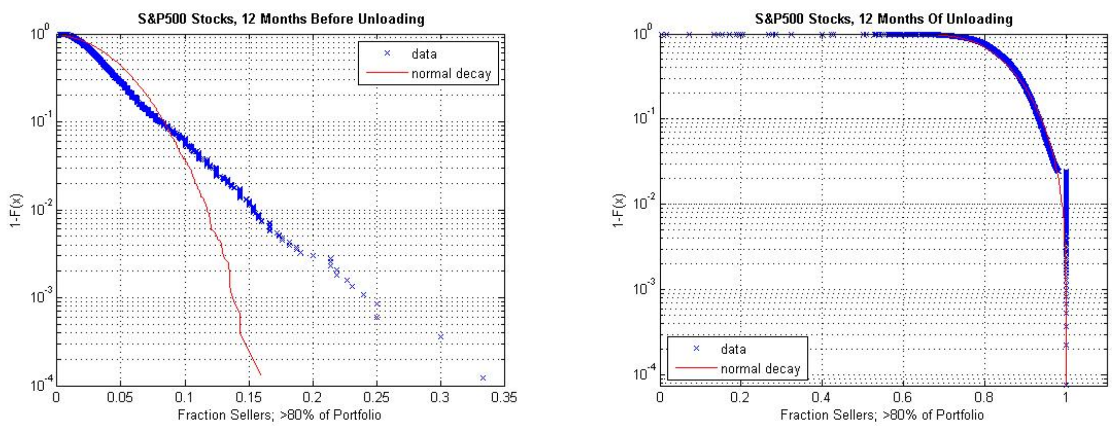

Figure 7: Semi-Log Probability Plots of $a(j,k)/N(j,k)$

The probability plot in the left panel also shows a convex deviation from the exponential tail as the size of observations approaches zero. This is consistent with a Gamma-type distribution, such as the distribution in Borel-Tanner family, which exhibits an exponential tail with a power decline near zero. Moreover, notice a small number of observations that lie very far from the probability mass. A Gamma-type distribution would produce such outliers because for small values of the shape parameter all observations drawn from a Gamma distribution will have the same expectation of the order $1/N$, but there is high probability that at least one observation will be much greater than the average (Kingman (1993)).

The intuition behind semi-log plots is as follows. Suppose the average perception of the value of fundamentals is strong and the mean fraction of institutional investment managers liquidating a particular stock is small. In the absence of selling cascades within some stock-investor-type groups the probability of observing a given value of $a(j,k)/N(j,k)$ would be declining at an increasing rate as we move further away from the mean. This Gaussian decay would produce a concave line in the semi-log plot. On the other hand, suppose investors are attempting to time the market by basing their actions on the actions of others. For example, within stock investor-type group a fund manager having observed a small fraction of other fund managers liquidating their holdings in a particular stock interprets this as the beginning of a "correction" and is induced to sell herself. If the conditions are so fragile that even in the absence of major changes in the fundamentals a number of investors are inclined to act as this hypothetical fund manager, then we would observe selling cascades within some stock-investor-type groups, creating outliers. Hence, if investment managers are locked in a herding regime then, even though the mean of the aggregate liquidation may still be low, the probability of observing large deviations from the mean will be higher than predicted by Gaussian decay that characterizes random deviations.

The right panel shows the semi-log probability plots of $a(j,k)/N(j,k))$ for 2006:Q2 through 2007:Q1. Consistent with transition from subcritical ($\mu < 1$) to supercritical phase ($\mu > 1$), this four quarter period is characterized by a state of "explosive" sell-outs. When the system is supercritical, there is a positive probability in which all the traders sell (explosion). Thus our model predicts a probability mass for fraction $a(j,k)/N(j,k) = 1$. If we allow for other randomness not considered in our model, then it is natural to think that the actual fraction is normally distributed around the mean close to 1.

The probability mass of $a(j,k)/N(j,k)$ is concentrated in the region between 0.8 and 1.0, indicating that the vast majority of institutional investors where dumping most of their S&P 500 stock. The relatively close fit of the normal distribution indicates that aggregate high mean value of $a(j,k)/N(j,k))$ is an informative summary statistic for the sell-out regime in the sense that the deviations from this high mean are random and the vast majority of institutions were liquidating their S&P 500 stocks during this period.

In sum, Figure 7 conveys two things. First, the sell-out ensued as early as 2006:Q2 and continued for approximately 4 quarters. Second, institutional investors in the stock market operated according to two different regimes during the duration of the bubble. During the run-up phase, the distribution of the aggregate action exhibits exponential decay, consistent with stochastic herding when the uncertainty over market timing actions of other institutional investment managers dominates. The exponential decay then vanishes during the sell-out phase. Such regime switching is consistent with transition from subcritical ($\mu < 1$) to supercritical phase ($\mu > 1$) with positive probability that all institutions act in unison (see *Theorem 1*).

Our hypothesis is that the process that generated empirical $a(j,k)$ shown in Figure 6 is best described probability density in Equation (22). We fit the model implied distribution against three alternatives: a truncated normal, Gamma, and Exponential. Table 9 shows the results.

The log likelihood values are higher for the model than any of the alternative distributions while truncated normal, which tests the possibility of Gaussian decay, has the smallest log likelihood value. In addition we conduct a non-nested goodness of fit test using Vuong's statistic. It is based on Kullback-Leibler information criterion which tests if the hypothesized models are equally close to the true model against the alternative that one is closer. Defining $l_i = logL(i;H_1) - logL(i;H_0)$ as the log likelihood ratio for each observation i, Vuong's statistic, $V \equiv (L_1 - L_0)/(\sqrt{N}Std(l_i))$, follows a standard normal distribution if the hypothesis H_0 and H_1 are equivalent. If $V > 1.96$ then H_0 of normal distribution is rejected in favor of H_1 of the model under 5% significance level. The Vuong statistics for the model (H_1) against H_0 that data follows either Gaussian, Gamma, or Exponential distributions are 30.393, 21.785, and 28.140 respectively rejecting H_0 in favor of the model.

Recall that μ_1 corresponds to the Poisson mean of the number of investors deciding to dump the stock when no one else is selling and μ quantifies that degree of herding which leads to a stretched tail in the distribution of $a(j,k)$. $\mu_1 = 2.068$ indicates approximately 2 managers within each group would have sold the stock even if no one else was selling. $\mu = 0.570$ indicates that on average during the 2005:Q2 through 2006:Q1 time period another fund manager would have chosen to follow the actions of these initial "random" sellers with a probability of 0.57.

4.5 Exponential Decay and the Rise of μ Over Time

Figure 10 through Figure 15 show quarterly semi-log probability plots of empirical $a(j,k)$ against the data simulated from the model and the two benchmark alternatives, Poisson and normal distributions. The data was simulated with distribution parameters first obtained via MLE using empirical $a(j,k)$.[12] A concave line corresponds to an accelerating probability decay in the tail characteristic of a Gaussian distribution while a straight line indicates decelerating exponential decay. The model of stochastic herding predicts that due to choice correlations the distribution of the

[12]Note that for 2003:Q4, 2004:Q1, 2004:Q4 we show a second plot with estimation dropping one outlier.

number of institutional investment managers liquidating a particular stock will exhibit exponential decay because of the persistency of outliers due to choice correlation.

During the early quarters (Figure 10), Poisson captures the probability decay close to the mean however misses the exponential decay in the tail. A normal distribution approximates the probability decline fairly well in 2003:Q2 when the probability mass in the empirical data follows a concave curve characteristic of the Gaussian decay. The fit of the model improves in 2003:Q4 and 2004:Q1, these are two quarters when mean and maximum of $a(j,k)$ temporarily increased (see Table 4). However, in both cases the empirical distribution exhibits bimodality and in both cases higher mean appears to have been driven by one outlier. It is nonetheless noteworthy that the tail of the distribution exhibits a rightward shift, as if pulled by the outliers but never lining up perfectly behind them.

The fit of the model improves substantially during 2006:Q1 (Figure 13), one quarter before the onset of the sell-off phase. The distribution of empirical $a(j,k)$ exhibits exponential decay, moreover the data points tend to from a more continuous line indicating higher instances of sell outs at intermediate values.

The following four quarters (2006:Q2 through 2007:Q1) the probability mass of $a(j,k)$ is concentrated around values an order of magnitude higher then in the previous period, indicating massive institutional dumping of stocks. Moreover, a more dense empirical plot indicates much greater incidence of $a(j,k)$ across all stock-investor type groups. However, during this period the distribution of $a(j,k)$ also exhibits bimodality, likely driven by heterogeneity in group sizes. This is because, as indicated in the discussion of Figure 7 in previous section, when controlling for group size via $a(j,k)/N(j,k)$, the bimodality disappears in favor of Gaussian decay around the mean close to 1.

After the sellout period the herding signature virtually vanishes – the empirical distribution of $a(j,k)$ is similar to the earlier periods of 2003 and 2004, with bimodal features (in 2007:Q2 and 2007:Q4 in particular) and the decay in the probability mass region approximated fairly well by a normal distribution.

Table 10 supplements graphical simulation analysis with quarterly MLE parameter estimates for the model. The last column shows the results of a non-nested goodness of fit test based on Vuong's statistic. If $V > 1.96$ then H_0 of normal distribution is rejected in favor of H_1 of the model under 5% significance level. The goodness of fit test confirms the inference made based on semilog probability plots and shows that the empirical distribution reject normal decay in favor of the model during all quarters except for 2006:Q2 through 2007:Q1. During the quarters when the

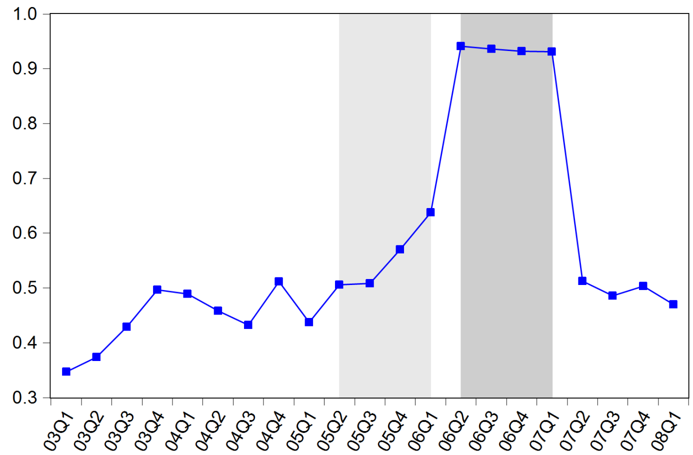

Figure 8: Herding – quarterly estimates of distribution parameter μ, which measures the probability of a "chain reaction" in response to a random liquidation by an investment manager. Initial independent liquidations occur with Poisson arrival rate of μ_1. The probability density of the aggregate action is then given by $\Pr(X = x) = \mu_1 e^{-(\mu x + \mu_1)}(\mu x + \mu_1)^{x-1}/x!$

model captures the empirical distribution the Poisson mean μ_1 is approximately 2 indicating that on average two investors in each stock-investor-type group, $N(j, k)$, chose to liquidated at random in the beginning of the tatonnement process. On the other hand, the estimates for μ, the degree of endogenous feedback, are rising from 0.347 in 2003:Q1 to 0.638 in 2006:Q1 indicating intensifying degree of herding up until sell-out phase.

The trend increase in μ, which accelerated during the last year before the sell-out phase, is shown in Figure 8. The estimate of μ in 2006:Q1 indicates that a random decision to dump the stock by an investment manager would have induced another investor to follow her action with a 64% probability. The rise of μ over time as the run-up on S&P 500 stocks continued is consistent with weakening fundamental anchors and a rising importance of market-timing considerations that make the system succeptible to herding. During the sell-out period the empirical data favors an alternate distribution, as seen by large negative Vuong's statistics. Note however that during the 2006:Q2 through 2007:Q1 period the estimates for μ range between 0.931 and 0.941 indicating that, although misspecified, the likelihood of a power-law with exponential truncation is maximized for μ close to 1, where $\mu = 1$ corresponds to the criticality at which exponential truncation vanished in favor of pure power law (consistent with semilog plots for this four quarter period shown in Figure 14). Finally, after the sellouts have subsided, exponential decay emerges once again but the estimates of μ remain below the 2006:Q1 level.

Overall, the rise of μ over time indicates that institutional investment manager actions increasingly exhibited contagious behavior intensifying the branching process until the sell-out phase. During the four quarters in 2003 the estimates of μ rise moderately after which point μ is approximately stationary until 2005:Q3, when μ begins to rise again until a sudden jump to the neighborhood of 1. This suggests that the population dynamics of fund manager behavior that we view as a the branching process with intensity μ transitioned from subcritical phase of $\mu < 1$ to a critical phase of $\mu = 1$ between 2006:Q1 and 2006:Q2. If in fact institutional fund managers learn about market liquidity, θ, by accumulating private information and observing aggregate action, then over time Bayesian learning ensures that beliefs about θ converge and the triggering action eventually occurs with probability 1.[13] This is because as private information, which is jointly normally distributed with the true θ hence informative, accumulates over time the average belief decreases causing some managers to liquidate even if no one else is liquidating. Their actions affect the threshold of others triggering a chain of liquidations. If sufficient amount of private information has been accumulated over time such that the average belief is low enough, then the chain reaction becomes "explosive" in the hence of self-organized criticality put forth by Bak et al. (1988). In Bak's sandpile model the distribution of the avalanche size depends on the slope of the sandpile. Our analog of the slope of the sandpile is the inverse of the average belief. At the criticality of $\mu = 1$ the distribution in Equation 22 becomes a pure power law and the branching process becomes a martingale, that is the conditional expectation then is that all managers liquidate next period if all are liquidating in the current period. Hence, then mean of $a(j, k)/N(j, k)$ approaching 1 in Table 5 sustained for four quarters and the symmetric distribution in the positive extreme of the histogram in the right panel Figure 5.

5 Conclusion

This paper has demonstrated that the behavior of institutional investors around the downturn of the U.S. equity markets in 2007 is consistent with stochastic herding in attempts to time the market. We considered a model of large number of institutional investment managers who simultaneously decide whether to remain invested in an assets or liquidate their positions. Each fund manager receives imperfect information about the market's ability to supply liquidity and chooses whether

[13] See Nirei (2006*a*) for a more general dynamic extension to information aggregation problem in financial markets.

or not to sell the security based on her private information as well as the actions of others. Because of feedback effects the equilibrium is stochastic and the "aggregate action" is characterized by a distribution exhibiting exponential decay embedding occasional "explosive" sell-outs. We can obtained such "fat tail" distributions without imposing major parametric assumptions on exogenous variables. It suffices that the signals about the true state are informative in the sense of satisfying the MLRP. For instance, as in this paper, the information and the true state can follow a bivariate normal distribution.

We examined highly disaggregated institutional ownership data of publicly traded stocks from 13F filings with SEC to find that stochastic herding explains the underlying data generating mechanism. Moreover, consistent with market-timing considerations, the distribution parameter measuring the degree of herding rose sharply immediately prior the sell-out phase that began in earnest in 2006:Q2. The transition to the sell-out itself is consistent with transition from subcritical to supercritical phase as the system swung sharply to a new equilibrium with all agents coordinating on the same action. One advantage of developing this empirical approach is its potential, given the right data, to quantify "hidden tail risk" and provide advance warning of an impeding instability by identifying a system with high degree of choice interdependence based on the distribution of aggregate action. These considerations should be important for both regulatory policy and risk management.

References

Abreu, D. and Brunnermeier, M. (2003), "Bubbles and crashes", *Econometrica* , Vol. 71, pp. 173–204.

Abreu, D. and Brunnermeier, M. K. (2002), "Synchronization risk and delayed arbitrage", *Journal of Financial Economics* , Vol. 66, pp. 341–360.

Alfarano, S. and Lux, T. (2007), "A noise trader model as a generator of apparent financial power laws and long memory", *Macroeconomic Dynamics* , Vol. 11, pp. 80–101.

Alfarano, S., Lux, T. and Wagner, F. (2005), "Estimation of agent-based models: The case of an asymmetric herding model", *Computational Economics* , Vol. 26, pp. 19–49.

Avery, C. and Zemsky, P. (1998), "Multidimensional uncertainty and herd behavior in financial markets", *American Economic Review* , Vol. 88, pp. 724–48.

Bak, P., Paczuski, M. and Shubik, M. (1997), "Price variations in a stock market with many agents", *Physica A: Statistical and Theoretical Physics* , Vol. 246, pp. 430 – 453.

Bak, P., Tang, C. and Wiesenfeld, K. (1988), "Self-organized criticality", *Phys. Rev. A* , Vol. 38, American Physical Society, pp. 364–374.

Banerjee, A. V. (1992), "A simple model of herd behavior", *The Quarterly Journal of Economics* , Vol. 107, pp. 797–817.

Bikhchandani, S., Hirshleifer, D. and Welch, I. (1992), "A theory of fads, fashion, custom, and cultural change in informational cascades", *Journal of Political Economy* , Vol. 100, pp. 992–1026.

Brunnermeier, M. K. (2009), "Deciphering the liquidity and credit crunch 2007-2008", *Journal of Economic Perspectives* , Vol. 23, pp. 77–100.

Brunnermeier, M. K. and Morgan, J. (2004), Clock games: Theory and experiments, Levine's bibliography, UCLA Department of Economics.

Brunnermeier, M. K. and Nagel, S. (2004), "Hedge funds and the technology bubble", *Journal of Finance* , Vol. 59, pp. 2013–2040.

Chari, V. V. and Kehoe, P. J. (2004), "Financial crises as herds: overturning the critiques", *Journal of Economic Theory* , Vol. 119, pp. 128–150.

Cheung, Y.-W. and Friedman, D. (2009), "Speculative attacks: A laboratory study in continuous time", *Journal of International Money and Finance* , Vol. 28, pp. 1064–1082.

Cowan, R. and Jonard, N. (2003), *Heterogeneous agents, Interactions and Economic Performance*, Lecture Notes in Economics and Mathematical Systems, Springer.

Fung, W. and Hsieh, D. A. (2000), "Measuring the market impact of hedge funds", *Journal of Empirical Finance* , Vol. 7, pp. 1–36.

Gabaix, X. (2008), Power laws in economics and finance, NBER Working Papers 14299, National Bureau of Economic Research, Inc.

Gabaix, X., Gopikrishnan, P., Plerou, V. and Stanley, H. E. (2006), "Institutional investors and stock market volatility", *The Quarterly Journal of Economics* , Vol. 121, pp. 461–504.

Gompers, P. and Metrick, A. (2001), "Institutional investors and equity prices", *The Quarterly Journal of Economics* , Vol. 116, pp. 229–259.

Gul, F. and Lundholm, R. (1995), "Endogenous timing and the clustering of agents' decisions", *Journal of Political Economy* , Vol. 103, pp. 1039–66.

Hardouvelis, G. and Stamatiou, T. (2009), Hedge funds and the us real estate bubble: Evidence from nyse real estate companies, University of piraeus working paper.

Jeon, J. Q. and Moffett, C. M. (2010), "Herding by foreign investors and emerging market equity returns: Evidence from korea", *International Review of Economics Finance* , Vol. 19, pp. 698 – 710.

Kim, K. A. and Nofsinger, J. R. (2005), "Institutional herding, business groups, and economic regimes: Evidence from japan", *Journal of Business* , Vol. 78, pp. 213–242.

Kingman, J. F. C. (1993), *Poisson Processes*, Oxford, NY.

Kirman, A. (1993), "Ants, rationality, and recruitment", *The Quarterly Journal of Economics* , Vol. 108, pp. 137–56.

Li, D. D. and Yung, K. (2004), "Institutional herding in the adr market", *Review of Quantitative Finance and Accounting* , Vol. 23, pp. 5–17.

McNichols, M. and Trueman, B. (1994), "Public disclosure, private information collection, and short-term trading", *Journal of Accounting and Economics* , Vol. 17, pp. 69–94.

Morris, S. and Shin, H. (1998), "Unique equilibrium in a model of self-fulfilling currency attacks", *American Economic Review* , Vol. 88, p. 587597.

Morris, S. and Shin, H. S. (1999), "Risk management with interdependent choice", *Oxford Review of Economic Policy* , Vol. 15, pp. 52–62.

Nirei, M. (2006*a*), Herd behavior and fat tails in financial markets, 2006 Meeting Papers 879, Society for Economic Dynamics.

Nirei, M. (2006*b*), "Threshold behavior and aggregate fluctuation", *Journal of Economic Theory* , Vol. 127, pp. 309–322.

Nirei, M. (2008), "Self-organized criticality in a herd behavior model of financial markets", *Journal of Economic Interaction and Coordination* , Vol. 3, pp. 89–97.

Nirei, M. and Sushko, V. (2011), "Jumps in foreign exchange rates and stochastic unwinding of carry trades", *International Review of Economics Finance, forthcoming* , Vol. 20, pp. 110 – 127.

Nofsinger, J. R. and Sias, R. W. (1999), "Herding and feedback trading by institutional and individual investors", *The Journal of Finance* , Vol. 54, pp. 2263 – 2295.

Rajan, R. G. (2006), "Has finance made the world riskier?", *European Financial Management* , Vol. 12, pp. 499–533.

Scharfstein, D. S. and Stein, J. C. (1990), "Herd behavior and investment", *American Economic Review* , Vol. 80, pp. 465–79.

Shiller, R. J. and Pound, J. (1989), "Survey evidence on diffusion of interest and information among investors", *Journal of Economic Behavior & Organization* , Vol. 12, pp. 47–66.

Shleifer, A., DeLong, J. B., Summers, L. H. and Waldmann, R. J. (1990), Noise trader risk in financial markets, J. Bradford DeLong's Working Papers 124.

Shleifer, A. and Vishny, R. W. (1997), "The limits of arbitrage", *Journal of Finance* , Vol. 52, pp. 35–55.

Sias, R. W. (2004), "Institutional herding", *Rev. Financ. Stud.* , Vol. 17, pp. 165–206.

Veldkamp, L. L. (2006), "Information markets and the comovement of asset prices", *Review of Economic Studies* , Vol. 73, pp. 823–845.

Welch, I. (2000), "Herding among security analysts", *Journal of Financial Economics* , Vol. 58, pp. 369–396.

*

A Proof of Proposition 1

First, we show that $\delta(\bar{x}, a)$ is increasing in a for a fixed value of $\bar{x}$. By completing the square on θ we obtain:

$$e^{-\frac{(x_i-\theta)^2}{2\sigma_e^2}} e^{-\frac{(\theta-\theta_0)^2}{2\sigma_0^2}} = e^{-\frac{\theta-\mu_\theta(x_i)}{2\sigma_\theta^2}} \xi(x_i) \tag{23}$$

where,

$$\mu_\theta(x_i) \equiv \frac{x_i/\sigma_e^2 + \theta_0/\sigma_0^2}{1/\sigma_e^2 + 1/\sigma_0^2} \tag{24}$$

$$\sigma_\theta^2 \equiv (1/\sigma_e^2 + 1/\sigma_0^2)^{-1} \tag{25}$$

$$\xi(x_i) \equiv \frac{\mu_\theta(x_i)^2}{2\sigma_\theta^2} - \frac{x_i^2}{2\sigma_e^2} - \frac{\theta_0^2}{2\sigma_0^2}. \tag{26}$$

Then we have:

$$\delta(x_i, a) = \frac{\int^\alpha e^{-\frac{(x_i-\theta)^2}{2\sigma_e^2}} e^{-\frac{(\theta-\theta_0)^2}{2\sigma_0^2}} d\theta}{\int_\alpha e^{-\frac{(x_i-\theta)^2}{2\sigma_e^2}} e^{-\frac{(\theta-\theta_0)^2}{2\sigma_0^2}} d\theta} = \frac{\Phi(\alpha; x_i)}{1 - \Phi(\alpha; x_i)} \tag{27}$$

where $\Phi(\cdot; x_i)$ denotes the cumulative distribution function for a normal distribution with mean $\mu_\theta(x_i)$ and variance σ_θ^2. When a is increased, the numerator rises and the denominator falls, and thus $\delta(x_i, a)$ increases.

Next we show that $A(\bar{x}, a)$ and $B(\bar{x}(k), a)$ increase in a for a fixed $\bar{x}$ and $k < a$. We start by showing that $G(\bar{x}, a)$ is increasing in the second argument:

$$\begin{aligned}
\frac{\partial G(\bar{x}, a)}{\partial a} &= \frac{\partial}{\partial a}\left(\frac{\Pr(x_j > \bar{x}, \theta < a/N)}{\Pr(\theta < a/N)}\right) && (28)\\
&= \frac{\Pr(x_j > \bar{x}, \theta = a/N)\Pr(\theta < a/N) - \Pr(x_j > \bar{x}, \theta < a/N)\Pr(\theta = a/N)}{\Pr(\theta < a/N)^2} && (29)\\
&= \frac{\Pr(\theta = a/N)}{\Pr(\theta < a/N)}\left(\frac{\Pr(x_j > \bar{x}, \theta = a/N)}{\Pr(\theta = a/N)} - \frac{\Pr(x_j > \bar{x}, \theta < a/N)}{\Pr(\theta < a/N)}\right) && (30)\\
&= \frac{\Pr(\theta = a/N)}{\Pr(\theta < a/N)}\left(\Pr(x_j > \bar{x} \mid \theta = a/N) - \Pr(x_j > \bar{x} \mid \theta < a/N)\right) && (31)\\
&> 0 && (32)
\end{aligned}$$

where "Pr" denotes likelihood functions. The last inequality holds by the property (14). We show likewise that $F(\bar{x}, a)$ is decreasing in a. Since $A(\bar{x}, a) = G(\bar{x}, a)/F(\bar{x}, a)$, we obtain that A is increasing in the second argument (a).

Finally, when a is increased by one, one trader switches sides from A to B, and this increases the right hand side of (13) because $A < B$. In sum, the right hand side increases in a for a fixed $\bar{x}$. Thus, if A is decreasing in $\bar{x}$, $\bar{x}(a)$ must be greater than $\bar{x}(a-1)$ in order to satisfy the equation (13).

Now we show that $\partial A/\partial \bar{x} < 0$. Define F_1 and G_1 as the derivatives of F and G with respect to the first argument $\bar{x}$, respectively. Then:

$$\frac{\partial A(\bar{x}, a)}{\partial \bar{x}} = \frac{F_1(\bar{x}, a)}{F(\bar{x}, a)}\left(\frac{G_1(\bar{x}, a)}{F_1(\bar{x}, a)} - A\right). \qquad (33)$$

G_1/F_1 can be rewritten as:

$$\frac{G_1(\bar{x}, a)}{F_1(\bar{x}, a)} = \frac{\Phi(\alpha; \bar{x})}{1 - \Phi(\alpha; \bar{x})}\frac{\Pr(\theta \geq \alpha)}{\Pr(\theta < \alpha)} \qquad (34)$$

A and B are written as:

$$\begin{aligned} A(\bar{x}, a) &= \frac{\int_{\bar{x}} \int^{\alpha} e^{-\frac{(x_i-\theta)^2}{2\sigma_e^2}} e^{-\frac{(\theta-\theta_0)^2}{2\sigma_0^2}} d\theta dx_i}{\int_{\bar{x}} \int_{\alpha} e^{-\frac{(x_i-\theta)^2}{2\sigma_e^2}} e^{-\frac{(\theta-\theta_0)^2}{2\sigma_0^2}} d\theta dx_i} \frac{\Pr(\theta \geq \alpha)}{\Pr(\theta < \alpha)} \\ &= \frac{\int_{\bar{x}} \Phi(\alpha; x_i) \xi(x_i) dx_i}{\int_{\bar{x}} (1 - \Phi(\alpha; x_i)) \xi(x_i) dx_i} \frac{\Pr(\theta \geq \alpha)}{\Pr(\theta < \alpha)} \qquad (35) \\ B(\bar{x}(k), a) &= \frac{\int^{\bar{x}(k)} \int^{\alpha} e^{-\frac{(x_i-\theta)^2}{2\sigma_e^2}} e^{-\frac{(\theta-\theta_0)^2}{2\sigma_0^2}} d\theta dx_i}{\int^{\bar{x}(k)} \int_{\alpha} e^{-\frac{(x_i-\theta)^2}{2\sigma_e^2}} e^{-\frac{(\theta-\theta_0)^2}{2\sigma_0^2}} d\theta dx_i} \frac{\Pr(\theta \geq \alpha)}{\Pr(\theta < \alpha)} \\ &= \frac{\int^{\bar{x}(k)} \Phi(\alpha; x_i) \xi(x_i) dx_i}{\int^{\bar{x}(k)} (1 - \Phi(\alpha; x_i)) \xi(x_i) dx_i} \frac{\Pr(\theta \geq \alpha)}{\Pr(\theta < \alpha)} \qquad (36) \end{aligned}$$

Then,

$$\frac{A}{G_1/F_1} = \int_{\bar{x}} \frac{\Phi(\alpha; x_i)}{\Phi(\alpha; \bar{x})} \xi(x_i) dx_i \Bigg/ \int_{\bar{x}} \frac{1 - \Phi(\alpha; x_i)}{1 - \Phi(\alpha; \bar{x})} \xi(x_i) dx_i. < 1 \qquad (37)$$

where the inequality obtains by that $\Phi(\alpha; x_i) < \Phi(\alpha; \bar{x})$ for any $x_i > \bar{x}$. Noting that $F_1 < 0$, we obtain from (33) that $\partial A(\bar{x}, a)/\partial \bar{x} < 0$.

B Proof of Proposition B

By taking logarithm of (13) for a and $a + 1$ and subtracting each side, we obtain:

$$\begin{aligned} 0 &= \log \delta(\bar{x}(a+1), a+1) - \log \delta(\bar{x}(a), a) + (N - 1 - a)(\log A(\bar{x}(a+1), a+1) - \log A(\bar{x}(a), a)) \\ &+ \sum_{k=0}^{a-1} (\log B(\bar{x}(k), a+1) - \log B(\bar{x}(k), a)) + \log B(\bar{x}(a), a+1) - \log A(\bar{x}(a+1), a+1) \qquad (38) \end{aligned}$$

The second argument a in δ and B affects the functions through $\alpha = a/N$ as in (27,36), and thus the direct effects of a' on δ and B are of order $1/N$. Also, as we show shortly, the difference $\bar{x}(a+1) - \bar{x}(a)$ is of order $1/N$, and so are a's effects through $\bar{x}$ on δ and B. Hence, the difference terms in (38) on $\log \delta$ and $\log B$ are of order $1/N$ and tends to zero as N goes to infinity.

The difference term in $\log A$ is broken down as:

$$\frac{\log A(\bar{x}(a+1), a+1) - \log A(\bar{x}(a), a)}{1/N} \sim_{N \to \infty} \frac{\partial \log A(\bar{x}(a), a)}{\partial \bar{x}} \frac{\bar{x}(a+1) - \bar{x}(a)}{1/N} + \frac{\partial \log A(\bar{x}(a), a)}{\partial a(1/N)} \qquad (39)$$

Thus, as $N \to \infty$ for a fixed finite a, we have:

$$(N-1-a)\left(\bar{x}(a+1)-\bar{x}(a)\right) \to \frac{\log B(\bar{x},a) - \log A(\bar{x},a) + \partial \log A(\bar{x}(a),a)/\partial\alpha}{-\partial \log A(\bar{x},a)/\partial \bar{x}} \qquad (40)$$

The right hand side is of order N^0, and hence it is shown that $\bar{x}(a+1)-\bar{x}(a)$ is of order $1/N$.

C Figures and Tables

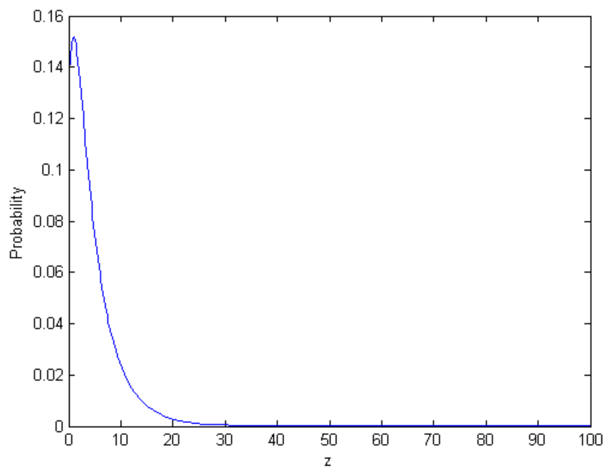

Figure 9: Probability density of the hypothesized distribution $\Pr(X = x) = \mu_1 e^{-(\mu x+\mu_1)}(\mu x + \mu_1)^{x-1}/x!$. Parameters estimated using 2005:Q2-2006:Q1 data on institutional investor holdings of S&P 500 stocks: $\mu_1 = 2.060$, $\mu = 0.547$.

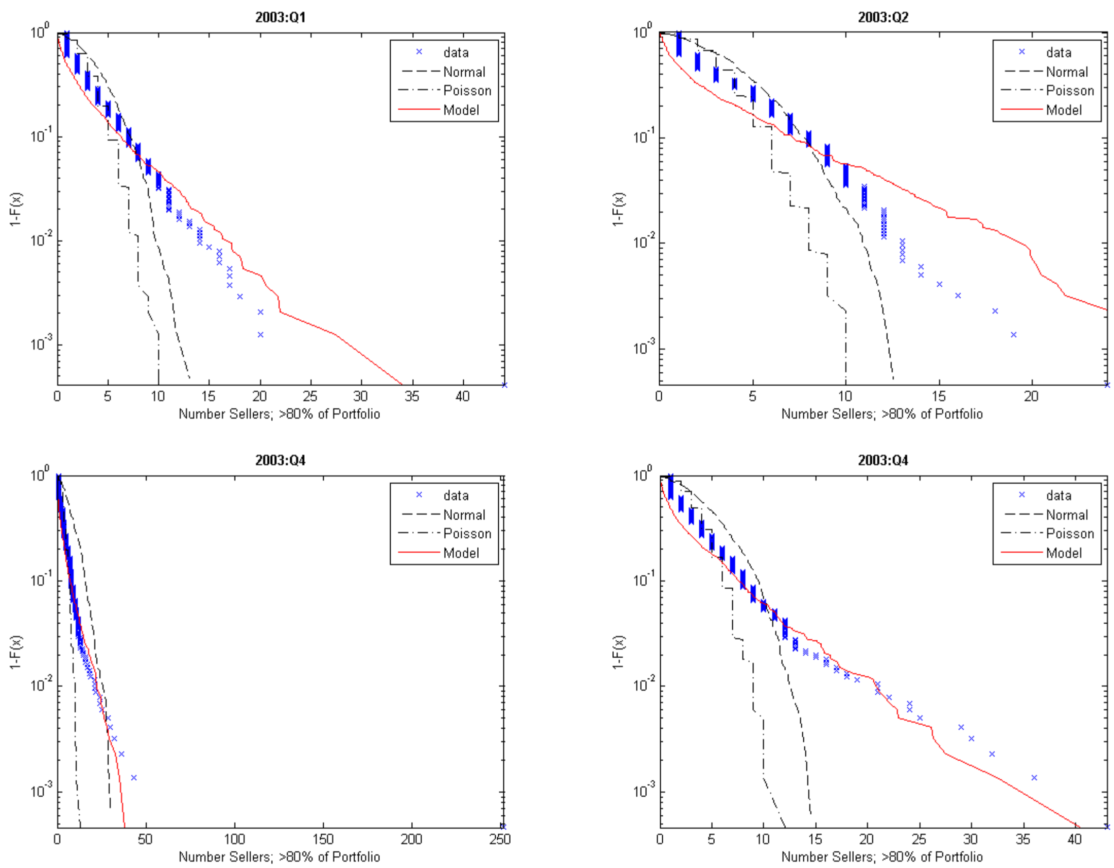

Figure 10: Semilog probability plot of $a(j,k)$ and comparison to data simulated using the model and the two alternatives, Normal and Poisson.

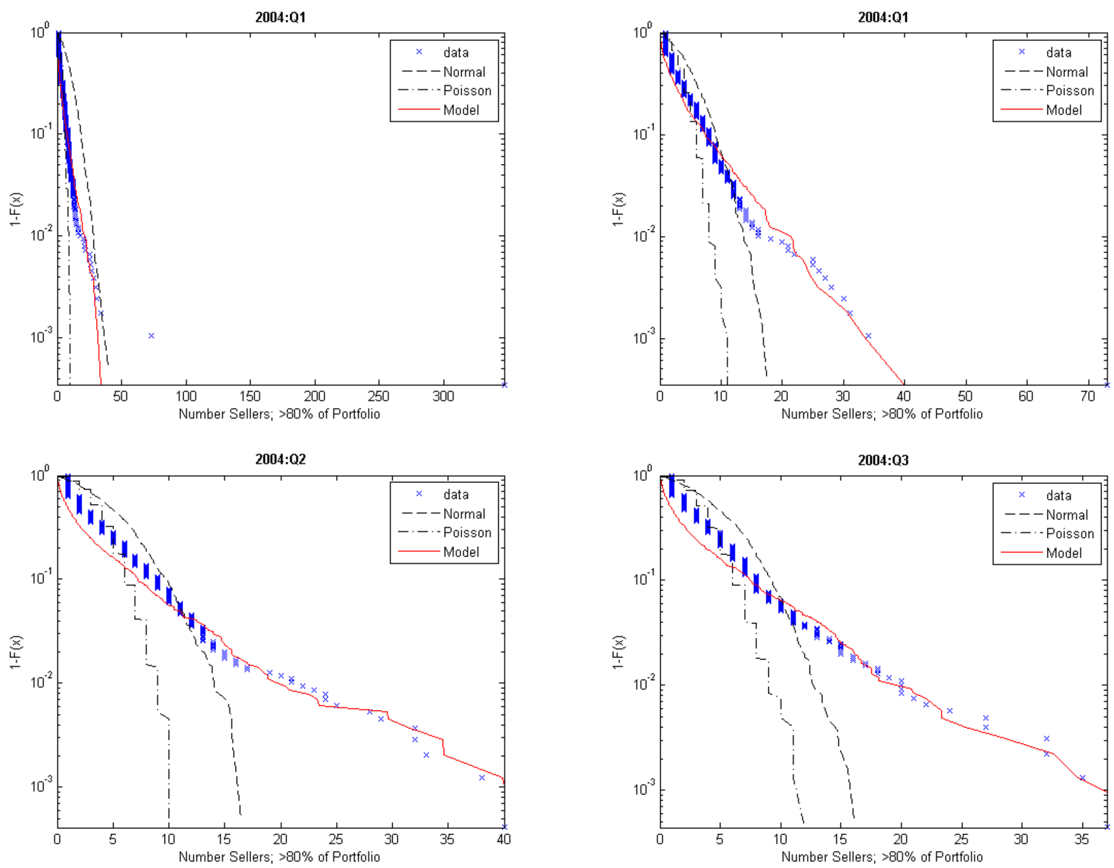

Figure 11: Semilog probability plot of $a(j,k)$ and comparison to data simulated using the model and the two alternatives, Normal and Poisson.

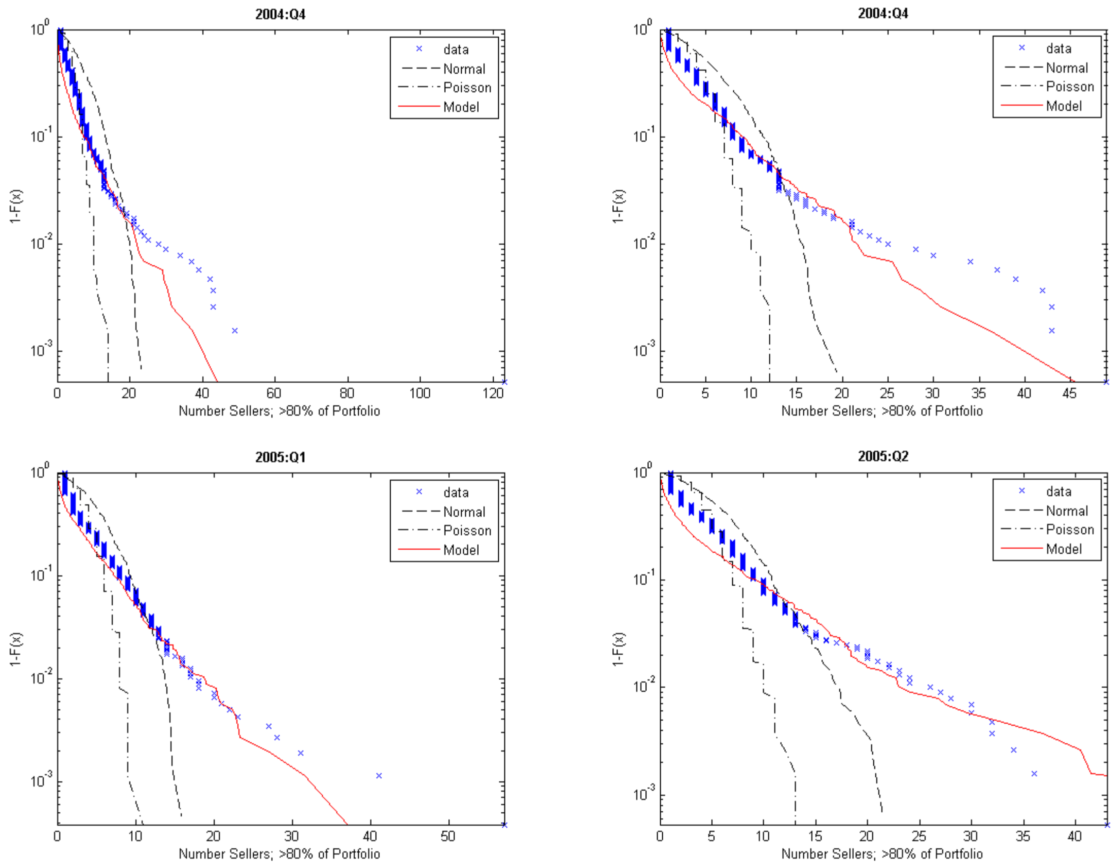

Figure 12: Semilog probability plot of $a(j,k)$ and comparison to data simulated using the model and the two alternatives, Normal and Poisson.

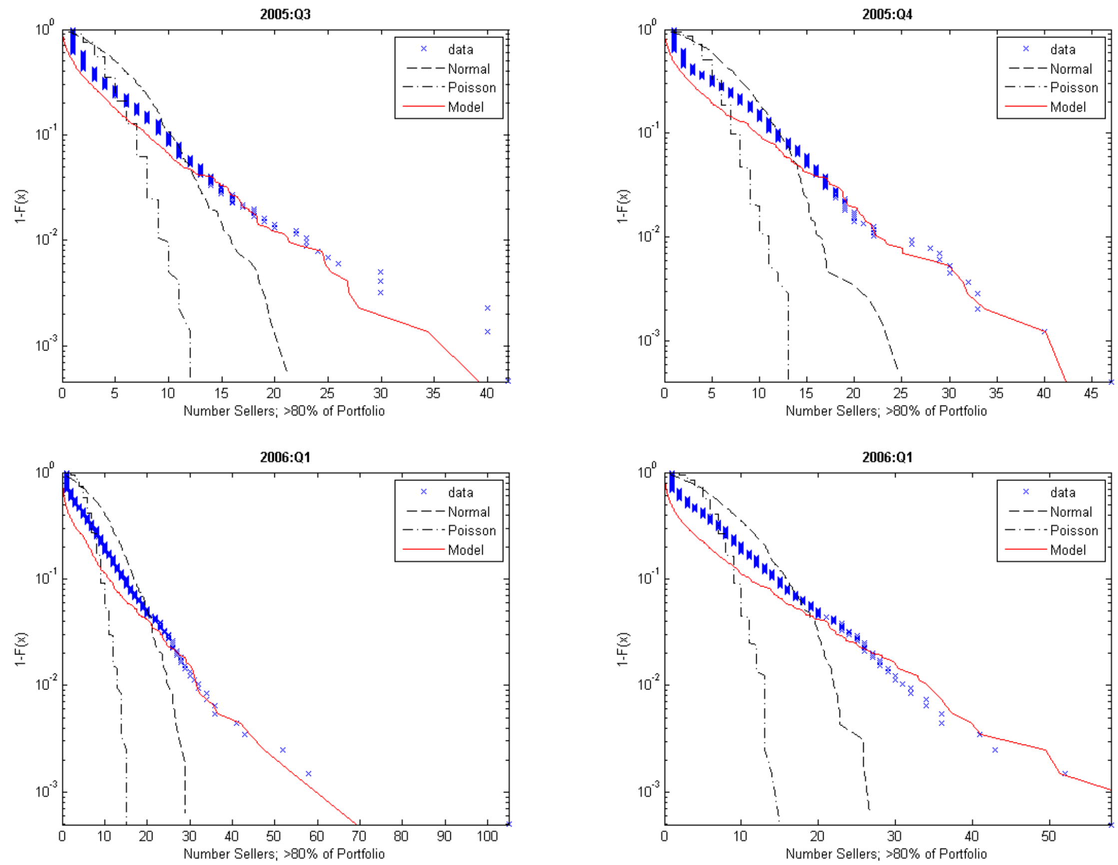

Figure 13: Semilog probability plot of $a(j,k)$ and comparison to data simulated using the model and the two alternatives, Normal and Poisson.

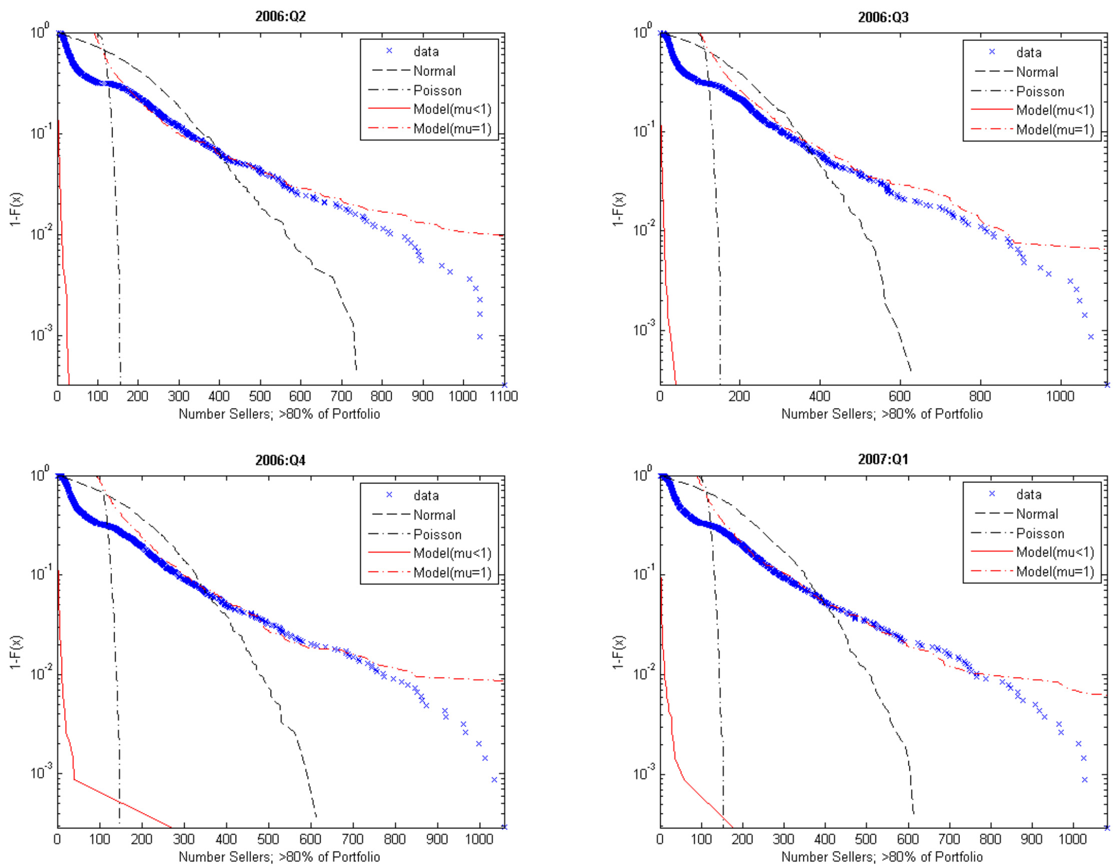

Figure 14: Semilog probability plot of $a(j,k)$ and comparison to data simulated using the model and the two alternatives, Normal and Poisson.

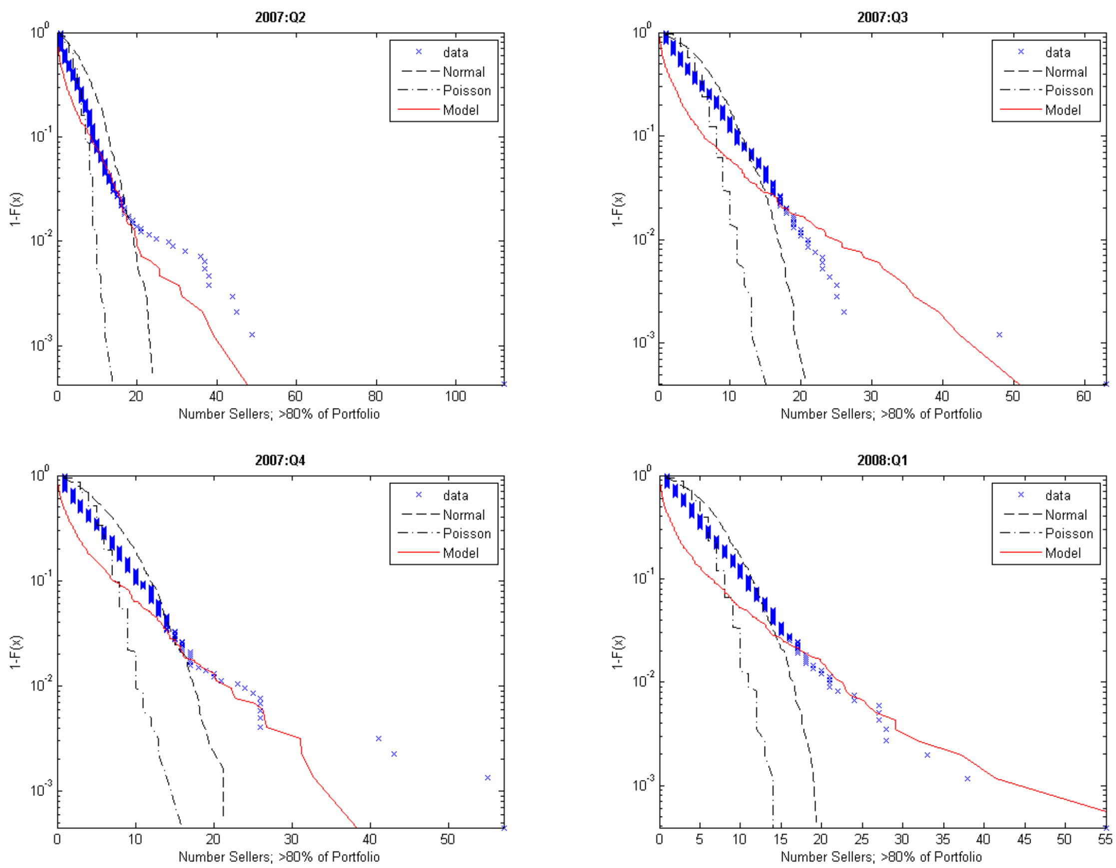

Figure 15: Semilog probability plot of $a(j,k)$ and comparison to data simulated using the model and the two alternatives, Normal and Poisson.

Quarter	Banks		Insurance Companies		Investment Companies		Investment Advisors		Pension & Endowment Funds		Total
	Number	% tot.	Number	% tot.	Number	% tot.	Number	% tot.	Number	% tot.	Number
2003q1	11	0.92	18	1.8	128	6.39	135	14.16	1579	76.74	1871
2003q2	11	0.87	19	1.98	130	6.42	137	14.16	1593	76.57	1890
2003q3	11	0.88	18	1.88	132	6.63	137	14.3	1590	76.31	1888
2003q4	10	1.01	21	1.71	121	5.95	131	13.74	1699	77.59	1982
2004q1	10	1.16	20	1.82	129	6.49	133	13.44	1742	77.09	2034
2004q2	10	0.99	20	1.81	133	13.29	136	6.4	1742	77.51	2041
2004q3	9	1.11	20	1.8	125	12.59	136	6.09	1735	78.41	2025
2004q4	10	1.2	20	1.68	117	11.44	166	6.97	1869	78.71	2182
2005q1	9	1.16	20	1.65	121	11.76	169	6.87	1877	78.56	2196
2005q2	8	1.08	20	1.64	119	11.22	170	6.73	1901	79.32	2218
2005q3	7	1.11	19	1.83	114	11.35	167	6.67	1855	79.04	2162
2005q4	8	0.96	19	1.62	114	10.96	187	7.18	1973	79.28	2301
2006q1	9	1.06	19	1.61	111	10.27	189	6.65	2024	80.41	2352
2006q2	10	1.16	18	1.59	106	9.9	204	7.3	2046	80.05	2384
2006q3	9	0.94	18	1.52	103	9.66	240	8.57	2014	79.3	2384
2006q4	9	1.12	17	1.4	102	9.43	333	11.98	2087	76.08	2548
2007q1	8	0.99	17	1.39	102	9.31	337	11.53	2124	76.78	2588
2007q2	10	0.97	18	1.38	101	8.84	384	13.47	2096	75.34	2609
2007q3	9	0.88	18	1.41	97	8.72	405	14.97	2058	74.01	2587
2007q4	9	0.96	18	1.34	97	8.34	516	17.54	2124	71.82	2764
2008q1	9	1.07	19	1.33	96	8.54	521	17.6	2119	71.41	2764

Table 1: Number of managers in S&P 500 stocks, by institution type

Quarter	Banks			Insur. Comp.			Invest. Comp.			Invest. Advisors			Pension & Endowment Funds			Total	
	Value ($ Mil.)	% Tot.	% Mkt.	Value ($ Mil.)	% Tot.	% Mkt.	Value ($ Mil.)	% Tot.	% Mkt.	Value ($ Mil.)	% Tot.	% Mkt.	Value ($ Mil.)	% Tot.	% Mkt.	Value ($ Mil.)	% Mkt.
2003q1	95,900	0.92	1.20	209,000	1.8	2.61	466,000	6.39	5.81	95,900	14.16	1.20	3,350,000	76.74	41.77	8,020,000	52.58
2003q2	108,000	0.87	1.19	235,000	1.98	2.58	541,000	6.42	5.94	1,080,000	14.16	11.86	3,860,000	76.57	42.37	9,110,000	63.93
2003q3	111,000	0.88	1.21	258,000	1.88	2.82	588,000	6.63	6.43	1,090,000	14.3	11.93	3,970,000	76.31	43.44	9,140,000	65.83
2003q4	76,000	1.71	0.75	128,000	1.01	1.27	207,000	5.95	2.05	1,280,000	13.74	12.67	4,370,000	77.59	43.27	10,100,000	60.01
2004q1	130,000	1.16	1.25	231,000	1.82	2.22	648,000	6.49	6.23	1,270,000	13.44	12.21	4,700,000	77.09	45.19	10,400,000	67.11
2004q2	129,000	0.99	1.19	220,000	1.81	2.04	648,000	6.4	6.00	1,260,000	13.29	11.67	4,700,000	77.51	43.52	10,800,000	64.42
2004q3	126,000	1.11	1.17	222,000	1.8	2.06	642,000	6.09	5.94	1,200,000	12.59	11.11	4,740,000	78.41	43.89	10,800,000	64.17
2004q4	137,000	1.2	1.17	235,000	1.68	2.01	737,000	6.97	0.00	1,270,000	11.44	10.85	5,220,000	78.71	44.62	11,700,000	58.65
2005q1	131,000	1.16	1.21	231,000	1.65	2.14	716,000	6.87	6.63	1,200,000	11.76	11.11	4,780,000	78.56	44.26	10,800,000	65.35
2005q2	130,000	1.08	1.19	231,000	1.64	2.12	718,000	6.73	6.59	1,160,000	11.22	10.64	4,920,000	79.32	45.14	10,900,000	65.68
2005q3	124,000	1.11	1.11	242,000	1.83	2.16	694,000	6.67	6.20	1,200,000	11.35	10.71	5,090,000	79.04	45.45	11,200,000	65.63
2005q4	130,000	0.96	1.14	246,000	1.62	2.16	724,000	7.18	6.35	1,220,000	10.96	10.70	5,360,000	79.28	47.02	11,400,000	67.37
2006q1	137,000	1.06	1.15	262,000	1.61	2.20	752,000	6.71	6.32	1,150,000	10.27	9.66	5,680,000	80.36	47.73	11,900,000	67.07
2006q2	144,000	1.16	1.17	264,000	1.59	2.15	778,000	7.28	6.33	1,170,000	9.9	9.51	5,860,000	80.07	47.64	12,300,000	66.80
2006q3	117,000	0.94	0.91	283,000	1.52	2.19	962,000	8.56	7.46	1,210,000	9.66	9.38	6,020,000	79.32	46.67	12,900,000	66.60
2006q4	159,000	1.12	1.18	297,000	1.4	2.20	1,430,000	11.96	10.59	1,490,000	9.43	11.04	5,820,000	76.09	43.11	13,500,000	68.12
2007q1	155,000	0.99	1.14	303,000	1.39	2.23	1,260,000	9.31	9.26	1,440,000	11.55	10.59	6,030,000	76.76	44.34	13,600,000	67.56
2007q2	160,000	0.97	1.13	369,000	1.38	2.60	1,300,000	8.84	9.15	1,860,000	13.47	13.10	6,120,000	75.34	43.10	14,200,000	69.08
2007q3	158,000	0.88	1.11	371,000	1.41	2.61	1,700,000	8.72	11.97	2,080,000	14.9	14.65	5,750,000	74.01	40.49	14,200,000	70.84
2007q4	165,000	0.96	1.22	351,000	1.34	2.60	1,350,000	8.34	10.00	2,130,000	17.54	15.78	5,330,000	71.82	39.48	13,500,000	69.08
2008q1	141,000	1.07	1.18	302,000	1.33	2.52	1,200,000	8.54	10.00	1,870,000	17.64	15.58	4,710,000	71.41	39.25	12,000,000	68.53

Table 2: Value of S&P 500 stocks; by institution type

Quarter	Obs.	Mean	Std. Dev.	Skewness	Kurtosis	Min.	Max.
2003q1	1842	114.2921	149.1265	2.559299	11.37729	10	1046
2003q2	1882	114.1265	150.8741	2.613972	11.82006	10	1105
2003q3	1856	115.715	150.2376	2.560607	11.42463	10	1069
2003q4	1846	118.792	157.0115	2.541808	11.26566	10	1130
2004q1	1878	121.2758	159.8353	2.527476	11.15328	10	1157
2004q2	1878	122.1081	161.6158	2.501442	10.95575	10	1150
2004q3	1859	119.2883	160.4453	2.509865	11.01269	10	1123
2004q4	1833	125.7239	168.1973	2.471413	10.65422	10	1145
2005q1	1546	136.6177	179.9863	2.328704	9.529656	10	1161
2005q2	1537	138.2492	184.3018	2.320885	9.460284	10	1187
2005q3	1562	133.0205	179.1581	2.332849	9.483129	10	1141
2005q4	1535	140.6098	186.0956	2.315662	9.374518	10	1170
2006q1	1543	142.4543	191.4796	2.259286	8.978757	10	1195
2006q2	1792	133.4492	182.3782	2.403041	10.02165	10	1205
2006q3	1788	133.9636	180.7052	2.45352	10.33914	10	1199
2006q4	1749	140.0743	177.5046	2.464914	10.48713	10	1197
2007q1	1720	143.0715	181.7965	2.401363	10.01951	10	1220
2007q2	1741	146.2211	181.5936	2.361122	9.723414	10	1222
2007q3	1738	141.2819	174.153	2.425098	10.18531	10	1179
2007q4	1689	148.2587	175.7674	2.368556	9.921134	10	1179
2008q1	1697	144.0595	171.5843	2.405454	10.21005	10	1197

Table 3: Descriptive Statistics: $N(j,k)$

Quarter	Obs.	Mean	Std. Dev.	Skewness	Kurtosis	Min.	Max.
2003q1	1842	2.024973	2.913459	3.562305	31.30286	0	44
2003q2	1882	2.001063	2.899889	2.022484	7.914702	0	24
2003q3	1856	2.199353	3.3151	1.87077	6.450509	0	23
2003q4	1846	2.299025	6.832766	26.99445	969.2564	0	252
2004q1	1878	2.78967	8.817813	32.14487	1240.28	0	347
2004q2	1878	2.457934	3.83505	3.523452	22.88747	0	40
2004q3	1859	2.257127	3.583172	3.511261	22.64714	0	37
2004q4	1833	2.240589	5.00605	10.326	202.7108	0	123
2005q1	1546	2.982536	3.905065	4.128196	37.58728	0	57
2005q2	1537	2.688354	4.341368	3.362496	20.00555	0	43
2005q3	1562	2.744558	4.306713	3.373875	20.45202	0	42
2005q4	1535	3.730945	5.142803	2.502752	12.18058	0	47
2006q1	1543	4.04731	6.772548	4.319921	42.25052	0	105
2006q2	1792	103.9738	160.4168	2.646657	11.52395	0	1100
2006q3	1788	115.1141	159.1397	2.650348	11.76884	0	1114
2006q4	1749	112.6781	150.5121	2.685956	12.0627	0	1057
2007q1	1720	116.8035	152.82	2.664223	11.87025	0	1080
2007q2	1741	3.036186	5.233333	7.706638	123.9169	0	112
2007q3	1738	3.659379	4.685157	3.009861	24.03532	0	63
2007q4	1689	3.117229	4.644519	3.760667	30.52305	0	57
2008q1	1697	3.727166	4.535413	2.734147	18.2615	0	55

Table 4: Descriptive Statistics: $a(j,k)$

Quarter	Obs.	Mean	Std. Dev.	Skewness	Kurtosis	Min.	Max.
2003q1	1203	0.0338277	0.0310166	2.800426	15.11922	0.0025974	0.2941177
2003q2	1090	0.03389	0.0274069	2.059337	8.368169	0.0026738	0.1904762
2003q3	985	0.032096	0.0296491	9.969194	182.9449	0.003861	0.6363636
2003q4	1088	0.0376633	0.0497596	8.49001	113.6885	0.000993	0.8113208
2004q1	1425	0.0398577	0.0462501	8.97107	134.2808	0.0027548	0.8202247
2004q2	1222	0.0392724	0.0350081	2.468958	13.05533	0.0023364	0.3636364
2004q3	1133	0.0378985	0.0329231	2.518009	14.81435	0.0027855	0.3571429
2004q4	961	0.0327997	0.0442431	8.801352	119.9139	0.0022422	0.7741935
2005q1	1308	0.0402617	0.031572	2.004466	8.523765	0.0023697	0.2666667
2005q2	947	0.0338203	0.0264428	2.175285	10.57713	0.0032626	0.2222222
2005q3	1088	0.034929	0.0301894	2.71143	14.75573	0.0022075	0.3043478
2005q4	1218	0.0407605	0.0281401	2.353076	13.41352	0.0033898	0.3
2006q1	1012	0.0416007	0.0283892	1.99926	8.37819	0.0035971	0.2
2006q2	1540	0.8455544	0.0936507	-1.105954	7.453066	0.137931	1
2006q3	1761	0.8789349	0.0704131	-1.133229	9.861254	0.1538462	1
2006q4	1731	0.7858961	0.0887199	-2.067533	13.76107	0.0185185	0.9473684
2007q1	1711	0.8308253	0.0812513	-3.080854	28.49091	0.0054054	1
2007q2	1180	0.0320758	0.0273514	2.807056	13.82265	0.0031315	0.2285714
2007q3	1253	0.0402934	0.0286195	1.766037	6.964115	0.0023256	0.2075472
2007q4	1110	0.030142	0.0215736	1.83541	7.408579	0.004329	0.1428571
2008q1	1280	0.0378383	0.0261516	1.951667	8.835462	0.0044643	0.2307692

Table 5: Descriptive Statistics: $a(j,k)/N(j,k)$

Variable	Obs.	Test Result	p-value	Test Stat.	Critical Value
$a(j,k)$	38,353	Reject	0.000	0.769	0.008

Table 6: Kolmogorov-Smirnov test, Poisson distribution of $a(j,k)$ over the entire sample, 2003:Q1 - 2008:Q1

Variable	Obs.	Mean	Std. Dev.	Skewness	Kurtosis
$a(j,k)$	38,353	22.745	79.264	6.368	54.868

Table 7: Test for normality of $a(j,k)$ over the entire sample, 2003:Q1 - 2008:Q1

Variable	Obs.	μ_1	μ	Log Likelihood
$a(j,k)$	38,353	2.058	0.938	99728.410
		(0.006)	(0.001)	

Table 8: Distribution parameter estimates for $a(j,k)$ for the entire sample, 2003:Q1 - 2008:Q1. The probability density for the hypothesized distribution is $\Pr(X = x) = \mu_1 e^{-(\mu x+\mu_1)}(\mu x + \mu_1)^{x-1}/x!$

	Distribution of $a(j,k)$							
	Model		Benchmark Distributions					
	Borel-Poisson		Trunc. Normal		Gamma		Exponential	
ML estimates	μ_1	2.058	mean	-97.461	α	1.103	β	4.781
		(0.029)		(7.152)		(0.021)		(0.072)
	μ	0.570	σ	20.000	β	4.335		
		(0.007)		(0.665)		(0.103)		
Log Likelihood		11148.789		10040.186		10925.596		10938.238
Vuong's statistic		H_1		30.393		21.785		28.140
Obs.		4,265						

Table 9: Distribution parameter estimates for $a(j,k)$ for the 2005:Q2 - 2006:Q1 subsample. The probability density for the hypothesized distribution is $\Pr(X = x) = \mu_1 e^{-(\mu x+\mu_1)}(\mu x + \mu_1)^{x-1}/x!$

Quarter	Obs.	μ_1	s.e.	μ	s.e.	Log Likelihood	Vuong's Statistic
2003q1	1203	2.023	(0.053)	0.347	(0.015)	2610.493	31.048
2003q2	1090	2.164	(0.060)	0.374	(0.016)	2479.326	28.628
2003q3	985	2.368	(0.069)	0.429	(0.016)	2412.327	25.413
2003q4	1088	1.963	(0.054)	0.497	(0.014)	2614.313	3.588
2004q1	1425	1.880	(0.045)	0.489	(0.012)	3343.738	–
2004q2	1222	2.046	(0.053)	0.458	(0.014)	2905.715	28.048
2004q3	1133	2.103	(0.057)	0.432	(0.014)	2661.898	28.686
2004q4	1833	2.087	(0.061)	0.512	(0.014)	2392.407	10.572
2005q1	1308	1.984	(0.050)	0.437	(0.013)	3024.613	23.576
2005q2	947	2.155	(0.064)	0.506	(0.015)	2384.011	22.919
2005q3	1088	1.938	(0.054)	0.508	(0.014)	2644.548	24.945
2005q4	1218	2.022	(0.054)	0.570	(0.012)	3170.367	25.764
2006q1	1012	2.234	(0.064)	0.638	(0.012)	2891.608	13.442
2006q2	1540	7.103	(0.138)	0.941	(0.002)	8660.686	-26.826
2006q3	1761	7.527	(0.136)	0.936	(0.002)	9866.110	-27.776
2006q4	1731	7.765	(0.142)	0.932	(0.002)	9696.680	-26.172
2007q1	1711	8.124	(0.149)	0.931	(0.002)	9645.581	-25.912
2007q2	1180	2.181	(0.057)	0.513	(0.013)	2991.997	15.100
2007q3	1253	2.606	(0.065)	0.486	(0.013)	3291.732	25.806
2007q4	1110	2.360	(0.064)	0.503	(0.013)	2867.704	24.548
2008q1	1280	2.619	(0.065)	0.470	(0.013)	3323.552	25.452

Table 10: Quarterly distribution parameter estimates for $a(j,k)$. The probability density for the hypothesized distribution is $\Pr(X = x) = \mu_1 e^{-(\mu x + \mu_1)}(\mu x + \mu_1)^{x-1}/x!$