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Makoto Nirei*

Institute of Innovation Research, Hitotsubashi University

2-1 Naka, Kunitachi, Tokyo 186-8603, Japan

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Abstract

This paper presents a model of endogenous fluctuations of investment and output at the business cycles frequencies. Aggregate investments fluctuate endogenously due to the strategic complementarity of micro-level lumpy investments. The investment fluctuations are transmitted to the output via variable utilization of capital. Simulations show that there is a range of parameter values under which the model economy exhibits a large magnitude of fluctuations and comovements in investment and output.

JEL Classification Codes: E32, E22

Keywords: Business cycles; lumpy investment; variable capacity utilization; nonlinear dynamics

*E-mail: nirei@iir.hit-u.ac.jp, Phone: +81 (42) 580-8417, Fax: +81 (42) 580-8410. I have benefitted by comments from Takashi Kano, Hashmat Khan, Tomoyuki Nakajima, Makoto Saito, John Stachurski, and Hajime Tomura.

1 Introduction

This paper presents a real dynamic general equilibrium model without exogenous shocks and with micro-level non-linearity, and shows that the equilibrium path can exhibit endogenous fluctuations of investment, output, and consumption at the business cycles frequencies.

This research is motivated by the fact that the standard real business cycle models need to assume large and persistent exogenous productivity shocks. It is still contentious whether such shocks are large and persistent enough as the theory requires (Cochrane (1994), Cogley and Nason (1995)). Various modifications are proposed to amend this problem: propagation mechanisms can render the assumed magnitude of exogenous shocks small (as discussed in King and Rebelo (1999)), and frictions can generate the persistent dynamics that resemble business cycles (for example, Christiano, Eichenbaum, and Evans (2005)). This paper presents an alternative approach for the amplification mechanism. We claim that the strategic complementarity of firm-level lumpy investments can generate aggregate fluctuations even without exogenous shocks.

In our model, the driving force of the fluctuation is the natural constant depreciation of capital. It is assumed that the investment is subject to discreteness constraint. Namely, each firm cannot adjust the capital level continuously, and it faces a binary decision whether it invests in a lumpy manner or not at all. If there were a continuum of firms, the aggregate dynamics would have a steady state level of capital, and the lumpiness would be "washed out". Namely, the fraction of firms that engage in lumpy investments times the lumpy investment would be equal to the amount of capital depreciation. In an economy with large but finite number of firms, there will be a gap between the total investment and the depreciated capital. The law of large numbers tells us that such a gap is vanishingly small for a large number of firms. This paper shows otherwise under certain environments where the small gap in investment is amplified by a strong propagation mechanism. We focus on the propagation that arises from the strategic complementarity of firms' lumpy investments in a monopolistically competitive economy. In principle, such a propagation mechanism can be mitigated by the adjustments of flexible factor prices (Thomas (2002)). We quantitatively show that substantial fluctuations can arise in a version of variable capital utilization model (Nakajima (2005)) for a range of parameter values.

The model has three ingredients that deviate from the benchmark competitive general equilibrium model: lumpy investments, monopolistic firms, and flexible but predetermined real interest rates. An explanation is due for the last point. The equilibrium condition of our model boils down to a non-linear dynamical system in very high dimensions – as many dimensions as the number of firms. Such a system generates deterministic complex dynamics for the aggregate variables. Even though the dynamics is deterministic, it is impossible to compute the exact path for many periods in future, because a small rounding error leads to a very different path of the capital profile. Thus, the model agents need to adopt some kind of computable forecasting system. We assume that the interest to be paid in period $t+1$ is determined in period t based on the forecasting system the agents employ. In addition to those specifications, we assume that a fraction of households follow a rule-of-thumb consumption/labor decision. This is to enhance the match with the fluctuations patterns of consumption.

We provide an analytical explanation as to why the micro-level lumpy investments can generate large endogenous fluctuations. The propagation effect of a firm's lumpy investment to the other firms depends sensitively on the distribution of the firms within the inaction band. It is clear that an investment boom occurs if many firms happen to be located near the investment threshold. It is not likely that the density near the threshold fluctuates very much though, because the cross-section distribution within the inaction band tends to converge to a stationary distribution quickly. We should note, however, that a slight difference in the density is enough to cause a chain-reaction of lumpy investments. The process is similar to a domino game. Suppose that a tile that is closest to the threshold falls. If the tile second closest to the threshold stands near enough to the first tile, it will fall, too. The second tile may similarly cause the third tile to fall, and the familiar domino effect ensues. The falling tiles stop where two adjacent tiles stand apart a little bit farther than the hight of the tile. Thus, a slight perturbation on the standing point is sufficient to cause a dramatic difference in the length of falling times. We will analyze the domino-like effect in our equilibrium model by investigating a fictitious tatonnement process that characterizes our equilibrium.

The next section introduces the model and equilibrium. Section 3 reports the numerical results. We discuss the endogenous fluctuation mechanism in Section 4. Section 5 concludes.

2 Model

2.1 Firm

The production sector of the model draws on Nakajima (2005). There are N firms, each of which produces a differentiated product denoted by $j = 1, 2, \ldots, N$. Firm j is a monopolistic producer of good j. Firm j has a within-period production function:

$$y_{j,t} = A_t u_{j,t} k_{j,t}^{\theta} \tag{1}$$

where $u_{j,t}$ is the capacity utilization rate that j can adjust within period t, and $k_{j,t}$ is the capital that is predetermined at the beginning of the period t. A_t is the aggregate productivity that is common across firms and grows at constant rate γ_A.

The capacity utilization rate $u_{j,t}$ is determined by the labor input $h_{j,t}$ as follows:

$$u_{j,t} = \frac{h_{j,t} - \underline{h}}{\bar{h} - \underline{h}} \tag{2}$$

This term is interpreted in Nakajima (2005) as follows. $\bar{h}$ represents the hours worked in a day when the production facility is in operation. $\underline{h}$ is the hours per day that are needed to maintain the facility when it is out of operation. $u_{j,t}$ is the fraction of time spent for operation in period t. Thus, the total labor input satisfies $h_{j,t} = u_{j,t}\bar{h} + (1 - u_{j,t})\underline{h}$. Solving for $u_{j,t}$, we obtain (2). The term $(1 - u_{j,t})\underline{h}$ is the fixed cost of the production. Due to the fixed cost, the production technology exhibits some increasing returns. The production function can be expressed as $y_{j,t} = (A_t/(\bar{h} - \underline{h}))(h_{j,t} - \underline{h})^{1-\theta}(u_{j,t}k_{j,t})^{\theta}$. Thus, this specification is consistent with the finding of Basu and Fernald (1995) when $\underline{h}$ is small enough.

Firm j faces a demand function for own good:

$$y_{j,t} = p_{j,t}^{-\eta} Y_t \tag{3}$$

where $Y_t = (\sum_{j=1}^{N} y_{j,t}^{(\eta-1)/\eta}/N)^{\eta/(\eta-1)}$ is the Dixit-Stiglitz index for aggregate product. The aggregate price index $P_t = (\sum_{j=1}^{N} p_{j,t}^{1-\eta}/N)^{1/(1-\eta)}$ is normalized to one. We assume $\eta > 1$, $\theta < 1$, and $\theta(\eta - 1) < 1$.

Finally, firm j's investment is restricted to a discrete choice, where the firm can choose either no gross investment, an upward jump, or a downward jump of capital by a lumpiness factor λ_j:

$$k_{j,t+1} \in \{\lambda_j(1 - \delta)k_{j,t},\ (1 - \delta)k_{j,t},\ (1 - \delta)k_{j,t}/\lambda_j\} \tag{4}$$

where $\delta \in (0, 1)$ is the capital depreciation rate. Such a discrete choice is observed when firms purchase big equipments or adjust the number of plants. The investment is denoted by $x_{j,t} = k_{j,t+1} - (1-\delta)k_{j,t}$. We assume $\lambda_j(1-\delta) > 1$. The investment is a Dixit-Stiglitz composite good, $x_{j,t} = (\sum_{i=1}^{N} x_{i,j,t}^{(\eta-1)/\eta}/N)^{\eta/(\eta-1)}$.

Firms are owned by forward-looking households and instructed to maximize the discounted sum of future profits by applying a discount factor Δ_t. Thus firm j's objective is:

$$\max_{\{y_{j,t}, p_{j,t}, h_{j,t}, u_{j,t}, k_{j,t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \Delta_t \left[p_{j,t} y_{j,t} - w_t h_{j,t} - k_{j,t+1} + (1-\delta)k_{j,t}\right] \tag{5}$$

subject to production function (1,2), demand function (3), and discreteness constraint (4).

Firm's first order condition with respect to $h_{j,t}$ leads to:

$$a_w y_{j,t}^{-1/\eta} Y_t^{1/\eta} A_t k_{j,t}^{\theta} = w_t \tag{6}$$

where

$$a_w \equiv (1 - 1/\eta)/(\bar{h} - \underline{h}). \tag{7}$$

From (6), we obtain a goods supply function:

$$y_{j,t} = (a_w A_t k_{j,t}^{\theta}/w_t)^{\eta} Y_t \tag{8}$$

Plugging into the production function (1,2), we obtain a labor demand function:

$$h_{j,t} = \underline{h} + (a_w A_t k_{j,t}^{\theta})^{\eta-1}(1-1/\eta) Y_t / w_t^{\eta} \tag{9}$$

Then the firm's maximization problem reduces to:

$$\max_{\{k_{j,t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \Delta_t \left[(1/\eta)(a_w A_t k_{j,t}^{\theta}/w_t)^{\eta-1} Y_t - w_t \underline{h} - k_{j,t+1} + (1-\delta)k_{j,t}\right] \tag{10}$$

subject to the discreteness constraint (4).

The part of the objective function that is relevant to the choice of $k_{j,t+1}$ is:

$$\pi(k_{j,t+1}) \equiv \Delta_{t+1}\left[(1/\eta)(a_w A_{t+1} k_{j,t+1}^{\theta}/w_{t+1})^{\eta-1} Y_{t+1} + (1-\delta)k_{j,t+1}\right] - \Delta_t k_{j,t+1}. \quad (11)$$

The function π is concave by the assumption $\theta(\eta-1) < 1$. Thus, the optimal strategy of firm j is characterized as an (S,s)-type threshold rule in which capital is not adjusted in $t+1$ if the depreciated capital $(1-\delta)k_t$ falls in an inaction region $(k_{j,t+1}^*, \lambda_j k_{j,t+1}^*]$, whereas capital is adjusted upward by λ_j in the region below $k_{j,t+1}^*$ and downward by $1/\lambda_j$ in the region above $\lambda_j k_{j,t+1}^*$. At the lower threshold $k_{j,t+1}^*$, the firm must be indifferent between an inaction and an upward adjustment. Note that, if the firm postpones the adjustment by one period and if the future adjustment plan is unchanged, then the capital path coincides with the original path from one period ahead. Thus, the firm is indifferent only if π is unchanged by the adjustment for the current period. Hence $k_{j,t+1}^*$ is determined by $\pi(k_{j,t+1}^*) = \pi(\lambda_j k_{j,t+1}^*)$. This is solved as follows:

$$k_{j,t+1}^* = a_j \left((1/\eta) a_w^{\eta-1} A_{t+1}^{\eta-1} w_{t+1}^{-(\eta-1)} (R_{t+1} - 1 + \delta)^{-1} Y_{t+1}\right)^{1/(1-\rho)} \quad (12)$$

where

$$\rho \equiv \theta(\eta - 1) < 1 \quad (13)$$

$$a_j \equiv \left[(\lambda_j^{\rho} - 1)/(\lambda_j - 1)\right]^{1/(1-\rho)} \quad (14)$$

$$R_{t+1} \equiv \Delta_t/\Delta_{t+1}. \quad (15)$$

Hence firm j's optimal policy is:

$$k_{j,t+1} = \begin{cases} \lambda_j (1-\delta) k_{j,t} & \text{if } (1-\delta)k_{j,t} \leq k^*_{j,t+1}, \\ (1-\delta)k_{j,t} & \text{if } k^*_{j,t+1} < (1-\delta)k_{j,t} \leq \lambda_j k^*_{j,t+1}, \\ (1-\delta)k_{j,t}/\lambda_j & \text{if } (1-\delta)k_{j,t} > \lambda_j k^*_{j,t+1}. \end{cases} \tag{16}$$

This completes the description of firm's optimal behavior.

2.2 Aggregation of firms' behaviors

By aggregating the goods supply function (8) across j, we obtain:

$$w_t = a_w A_t K_t^{\theta} \tag{17}$$

where $K_t \equiv \sum_{j=1}^{N} (k_{j,t}^{\rho}/N)^{1/\rho}$. Equation (17) determines the equilibrium wage at the marginal product of labor given the aggregate capital level. The supply function for good j reduces to $y_{j,t} = (k_{j,t}/K_t)^{\theta} Y_t$. Aggregating this supply function, we observe that the aggregate supply is indeterminate within the firms' sector. This is because the within-period production is linear in labor. The equilibrium level of aggregate production is thus determined by consumption and investment demands.

Plugging into (12), we obtain the optimal threshold rule for firm j as follows:

$$k^*_{j,t+1} = a_j \left((1/\eta) K_{t+1}^{-\rho} (R_{t+1} - 1 + \delta)^{-1} Y_{t+1} \right)^{1/(1-\rho)}. \tag{18}$$

The aggregate labor demand function is obtained by summing the individual demand functions (9) across firms. Using the equilibrium relation (17), we obtain:

$$H_t \equiv \sum_{j=1}^{N} h_{j,t}/N = \underline{h} + \left(1 - \frac{1}{\eta}\right) \frac{Y_t}{w_t}. \tag{19}$$

Define a firm's state variable in the inaction region as:

$$s_{j,t} \equiv \frac{\log k_{j,t} - \log k^*_{j,t}}{\log \lambda_j}. \tag{20}$$

Under the (S,s) rule (16), $s_{j,t}$ takes values in the unit interval. Let $z_{j,t}$ denote the threshold of $s_{j,t}$ below which the optimal action is to increase capital in the next period $t+1$. That is, if we start from $s_{j,t} = z_{j,t}$, then $s_{j,t+1} = 0$ holds for $k_{j,t+1} = (1-\delta)k_{j,t}$. Thus,

$$0 = s_{j,t+1} = (\log k_{j,t+1} - \log k^*_{j,t+1})/\log \lambda_j \tag{21}$$

$$= (\log(1-\delta) + \log k_{j,t} - \log k^*_{j,t+1})/\log \lambda_j \tag{22}$$

$$= z_{j,t} + (\log(1-\delta) - \log k^*_{j,t+1} + \log k^*_{j,t})/\log \lambda_j. \tag{23}$$

Then,

$$z_{j,t} = \frac{1}{q_j} + \frac{\log k^*_{j,t+1} - \log \gamma - \log k^*_{j,t}}{\log \lambda_j} \tag{24}$$

where

$$q_j \equiv \frac{\log \lambda_j}{\log \gamma + |\log(1-\delta)|} \tag{25}$$

is the natural frequency of capital adjustments for firm j, where $\log \gamma$ is the trend growth rate of output and capital.

No firms adjust capital downward (other than depreciation) when the economy is around the stationary level, if the lumpiness $\log \lambda_j$ is sufficiently larger than the depreciation $|\log(1-\delta)|$. Then, the law of motion for aggregate capital is written as:

$$K_{t+1} = \left(\left(\sum_{\{j: s_{j,t} > z_{j,t}\}} ((1-\delta)k_{j,t})^\rho + \sum_{\{j: s_{j,t} \leq z_{j,t}\}} (\lambda_j (1-\delta) k_{j,t})^\rho \right) /N \right)^{1/\rho}. \tag{26}$$

Thus,

$$K_{t+1}^{\rho} = (1-\delta)^{\rho}\left(\sum_{j=1}^{N} k_{j,t}^{\rho} + \sum_{\{j:s_{j,t}\leq z_{j,t}\}} (\lambda_j^{\rho}-1)k_{j,t}^{\rho}\right)/N \tag{27}$$

$$= (1-\delta)^{\rho}\left(K_t^{\rho} + \sum_{\{j:s_{j,t}\leq z_{j,t}\}} (\lambda_j^{\rho}-1)k_{j,t}^{\rho}/N\right). \tag{28}$$

Similarly, the aggregate investment is expressed as follows:

$$X_t = \sum_{j=1}^{N} x_{j,t}/N = (1-\delta)\sum_{\{j:s_{j,t}\leq z_{j,t}\}} (\lambda_j-1)k_{j,t}/N. \tag{29}$$

2.3 Households

We assume that there are rational households and rule-of-thumb households. $(1-\chi)$ fraction of households follow a rule-of-thumb on their consumption and labor decision. Their labor supply is set equal to the average in the economy. Namely, the rule-of-thumb household's labor supply follows $H_{ROT,t} = H_t$. They also consume all the wage income less tax payment, and do not hold any asset. Hence $C_{ROT,t} = w_t H_{ROT,t} - T_t$ where T_t is the lump-sum tax payment. The other, χ fraction of households are rational, forward-looking decision makers, and they own the firms. The rational households have a momentary utility $\log C_{R,t} - H_{R,t}$ and a flow budget constraint $C_{R,t} = w_t H_{R,t} + \Pi_t/\chi - T_t$, where Π_t is the profit from firms. Thus the first order condition for contemporaneous consumption yields:

$$C_{R,t} = w_t. \tag{30}$$

The rational households discount the future utility by β. In each period t, the rational households instruct firms about their marginal rate of intertemporal substitution, which the firms use as a discount factor in maximizing their values. If the rational households know the future states, then the discount factor would be

$R_{t+1} = w_{t+1}/(w_t\beta) = (A_{t+1}/A_t)(K_{t+1}/K_t)^\theta$. We assume that the rational households know the realization of the future aggregate productivity A_{t+1}, but that they do not know the realization of the future aggregate capital K_{t+1} in t. We consider the case where households know the current aggregate capital K_t but do not have a precise information on its configuration across firms, $(k_{j,t})$. Then households are assumed to form an expected aggregate capital K^e_{t+1} by approximating $(k_{j,t})$ by its stationary counterpart, as we formulate in detail shortly. Thus the discount factor instructed by the rational households follows:

$$R_{t+1} = \frac{w^e_{t+1}}{\beta w_t} = \frac{A_{t+1}}{\beta A_t}\left(\frac{K^e_{t+1}}{K_t}\right)^\theta. \tag{31}$$

The aggregation relation $C_t = (1-\chi)C_{ROT,t} + \chi C_{R,t}$ must hold. Combining with the aggregate labor demand (19) and (30), we obtain the aggregate consumption demand as a function of wage and income:

$$C_t = b_w w_t + (1-a_y)Y_t - (1-\chi)T_t \tag{32}$$

where

$$b_w \equiv \chi + (1-\chi)\underline{h} \tag{33}$$

$$a_y \equiv 1-(1-\chi)(1-1/\eta). \tag{34}$$

Finally, there is a government who collects lump-sum tax T_t and spends the proceed on purchase of goods G_t. The government's budget is always balanced: $G_t = T_t$ for all t. We also assume that the government purchase G_t grows constantly at the same rate as the trend growth of Y_t. Thus, the detrended government purchase is a constant fraction τ of the steady-state level of the detrended output ($\bar{Y}$ as we define shortly).

2.4 Equilibrium

The goods and labor markets must clear at equilibrium. Hence,

$$Y_t = C_t + X_t + G_t \tag{35}$$

$$H_t = (1-\chi)H_{ROT,t} + \chi H_{R,t} \tag{36}$$

where $X_t \equiv \sum_{j=1}^{N} x_{j,t}/N$ is the aggregate investment. $H_{R,t} = H_t$ immediately follows (36). Combining the market clearing conditions (35) and the consumption function (32), we obtain:

$$a_y Y_t = b_w w_t + X_t + \chi G_t. \tag{37}$$

Consumption $C_{R,t}$ and $C_{ROT,t}$ are Dixit-Stiglitz composite goods: $C_{R,t} = (\sum_{i=1}^{N} c_{R,i,t}^{(\eta-1)/\eta}/N)^{\eta/(\eta-1)}$ and $C_{ROT,t} = (\sum_{i=1}^{N} c_{ROT,i,t}^{(\eta-1)/\eta}/N)^{\eta/(\eta-1)}$. We assume that the rule-of-thumb households do minimize cost when they purchase $c_{ROT,i,t}$, just like the rational households do. Then the derived demand for $c_{R,i,t}$, $c_{ROT,i,t}$, and $x_{j,i,t}$ are solved given $C_{R,t}$, $C_{ROT,t}$, and $x_{j,t}$. By the usual procedure with Dixit-Stiglitz indices, the market clearing conditions for individual goods aggregate up to the market clearing condition for composite goods (35). Then, the aggregation of derived demand $x_{i,j,t}$, ($c_{R,j,t}$, $c_{ROT,j,t}$) across i's and households yields the demand function for individual good j, as we supposed in (3).

A perfect foresight equilibrium is the price system $(p_{j,t}, w_t, R_t)$ and allocation

$$(C_t, C_{R,t}, C_{ROT,t}, c_{R,i,t}, c_{ROT,i,t}, H_t, H_{R,t}, H_{ROT,t}, h_{j,t}, K_{t+1}, k_{j,t+1}, X_t, x_{j,t}, x_{j,i,t}, Y_t, y_{j,t}, u_{j,t})$$

such that the allocation solves the firm's optimization problem and the rational household's problem under the prices and that the allocation satisfies the rules of the government and the rule-of-thumb households and clears the goods and labor markets. Summarizing the conditions derived above, the perfect foresight equilibrium path is determined by the following system of equations:

$$w_t = a_w A_t K_t^\theta \tag{38}$$

$$R_{t+1} = \frac{A_{t+1}}{\beta A_t} \left(\frac{K_{t+1}^e}{K_t} \right)^\theta \tag{39}$$

$$K_{t+1}^e = K_{t+1} \tag{40}$$

$$a_y Y_t = b_w w_t + X_t + \chi G_t \tag{41}$$

$$K_{t+1}^\rho = (1-\delta)^\rho \left(K_t^\rho + \sum_{\{j: s_{j,t} \le z_{j,t}\}} (\lambda_j^\rho - 1) k_{j,t}^\rho / N \right) \tag{42}$$

$$X_t = (1-\delta) \sum_{\{j: s_{j,t} < z_{j,t}\}} (\lambda_j - 1) k_{j,t} / N \tag{43}$$

$$s_{j,t} = \frac{\log k_{j,t} - \log k_{j,t}^*}{\log \lambda_j} \tag{44}$$

$$z_{j,t} = \frac{1}{q_j} + \frac{\log k_{j,t+1}^* - \log k_{j,t}^*}{\log \lambda_j} \tag{45}$$

$$k_{j,t}^* = a_j \left((1/\eta) K_t^{-\rho} (R_t - 1 + \delta)^{-1} Y_t \right)^{1/(1-\rho)} \tag{46}$$

2.5 Approximation of the future equilibrium path

X_t is very sensitive on detailed configurations of the capital profile k_t, as we argue in Section 4. Due to the lumpiness of investments, the capital dynamics follows a high-dimensional non-linear system which leads to a chaotic path of the aggregate capital. Therefore, it is practically impossible for agents to compute the perfect foresight equilibrium path without employing some approximations to forecast the future equilibrium path. We assume that the agents approximate the cross-section distribution of $s_{j,t}$ by a uniform distribution of a continuum of firms over the unit interval. It turns out that this is a very good approximation of the actual distribution of $s_{j,t}$. In fact, it has been emphasized in the literature that a general one-sided (S,s) economy has a robust tendency in which $s_{j,t}$ converges to the uniform distribution (Caplin and Spulber (1987), Caballero and Engel (1991)).

By replacing $(s_{j,t})$ with a continuum of uniformly distributed random variables, (28) is modified as follows.

$$
\begin{aligned}
K_{t+1}^{\rho} &= (1-\delta)^{\rho}\left(K_t^{\rho} + \sum_{\{j:s_{j,t}\le z_{j,t}\}} (\lambda_j^{\rho}-1)k_{j,t}^{\rho}/N\right) && (47)\\
&= (1-\delta)^{\rho}\left(\sum_{j=1}^{N} \lambda_j^{s_{j,t}\rho} k_{j,t}^{*\rho} + \sum_{\{j:s_{j,t}\le z_{j,t}\}} (\lambda_j^{\rho}-1)\lambda_j^{s_{j,t}\rho} k_{j,t}^{*\rho}\right)/N && (48)\\
&\approx (1-\delta)^{\rho} E\left(\int_0^1 \lambda_j^{s_{j,t}\rho} k_{j,t}^{*\rho} ds_{j,t} + \int_0^{z_{j,t}} (\lambda_j^{\rho}-1)\lambda_j^{s_{j,t}\rho} k_{j,t}^{*\rho} ds_{j,t}\right) && (49)\\
&= (1-\delta)^{\rho} E\left(\left(\frac{\lambda_j^{\rho}-1}{\rho\log\lambda_j} + (\lambda_j^{\rho}-1)\frac{\lambda_j^{z_{j,t}\rho}-1}{\rho\log\lambda_j}\right) k_{j,t}^{*\rho}\right) && (50)\\
&= (1-\delta)^{\rho} E\left(\frac{\lambda_j^{\rho}-1}{\rho\log\lambda_j}\lambda_j^{z_{j,t}\rho} k_{j,t}^{*\rho}\right) && (51)\\
&= (1-\delta)^{\rho} E\left(\frac{\lambda_j^{\rho}-1}{\rho\log\lambda_j}\lambda_j^{\rho/q_j} (k_{j,t+1}^{*}/k_{j,t}^{*})^{\rho} k_{j,t}^{*\rho}\right) && (52)\\
&= E\left(\frac{\lambda_j^{\rho}-1}{\rho\log\lambda_j} a_j^{\rho}\right)\left((1/\eta)K_{t+1}^{-\rho}(R_{t+1}-1+\delta)^{-1}Y_{t+1}\right)^{\rho/(1-\rho)} && (53)
\end{aligned}
$$

where the expectation is taken across heterogenous λ_j and a_j. In the manipulation, we used the facts that $\lambda_j^{z_{j,t}} = \lambda_j^{1/q_j} k_{j,t+1}^{*}/k_{j,t}^{*}$ and $\lambda_j^{1/q_j} = (1-\delta)^{-1}$. Then we get:

$$K_{t+1} \approx (a^{1-\rho}/\eta)(R_{t+1}-1+\delta)^{-1}Y_{t+1} \tag{54}$$

where

$$a \equiv E\left(\frac{\lambda_j^{\rho}-1}{\rho\log\lambda_j} a_j^{\rho}\right)^{1/\rho}. \tag{55}$$

Similarly, (29) is rewritten as follows under the continuum of uniformly distributed

$s_{j,t}$:

$$
\begin{aligned}
X_t &= (1-\delta) \sum_{\{j: s_{j,t} < z_{j,t}\}} (\lambda_j - 1) k_{j,t}/N && (56)\\
&= (1-\delta) \sum_{\{j: s_{j,t} < z_{j,t}\}} (\lambda_j - 1) \lambda_j^{s_{j,t}} k^*_{j,t}/N && (57)\\
&= (1-\delta) E\left(\int_0^{z_{j,t}} (\lambda_j - 1) \lambda_j^{s_{j,t}} ds_{j,t} k^*_{j,t} \right) && (58)\\
&= (1-\delta) E\left((\lambda_j - 1) \frac{\lambda_j^{z_{j,t}} - 1}{\log \lambda_j} k^*_{j,t} \right) && (59)\\
&= (1-\delta) E\left(\frac{\lambda_j - 1}{\log \lambda_j} \left(\lambda_j^{1/q_j} k^*_{j,t+1}/k^*_{j,t} - 1 \right) k^*_{j,t} \right) && (60)\\
&= (1-\delta) E\left(\frac{\lambda_j - 1}{\log \lambda_j} \left((1-\delta)^{-1} k^*_{j,t+1} - k^*_{j,t} \right) \right) && (61)\\
&= E\left(\frac{\lambda_j - 1}{\log \lambda_j} \left(k^*_{j,t+1} - (1-\delta) k^*_{j,t} \right) \right) && (62)\\
&= E\left(\frac{\lambda_j - 1}{\log \lambda_j} \frac{a_j}{a} (K_{t+1} - (1-\delta) K_t) \right). && (63)
\end{aligned}
$$

Thus,

$$
X_t \approx c_X (K_{t+1} - (1-\delta) K_t) \tag{64}
$$

where,

$$
c_X \equiv E\left(\frac{\lambda_j - 1}{\log \lambda_j} \frac{a_j}{a} \right). \tag{65}
$$

The approximated dynamics under the continuum approximation is summarized as follows:

$$
\begin{aligned}
K_{t+1} &= \frac{a^{1-\rho}}{\eta} \left(\frac{w_{t+1}}{\beta w_t} - 1 + \delta \right)^{-1} Y_{t+1} && (66)\\
w_t &= a_w A_t K_t^{\theta} && (67)\\
a_y Y_t &= b_w w_t + c_X (K_{t+1} - (1-\delta) K_t) + \chi G_t. && (68)
\end{aligned}
$$

The system boils down to a second-order difference equation in K_t:

$$\left(\frac{A_{t+1}}{\beta A_t}\left(\frac{K_{t+1}}{K_t}\right)^{\theta}-1+\delta\right)K_{t+1}=\frac{a^{1-\rho}}{\eta a_y}\left(b_w a_w A_{t+1}K_{t+1}^{\theta}+c_X(K_{t+2}-(1-\delta)K_{t+1})+\chi G_{t+1}\right). \tag{69}$$

The aggregate productivity has a constant growth rate $\log\gamma_A$. Thus, the output has a time trend $\log\gamma\equiv\log\gamma_A/(1-\theta)$. We normalize variables by the growth factor and denote them by a hat. Namely, $\hat{K}_t\equiv K_t/A_t^{1/(1-\theta)}$, $\hat{w}_t\equiv w_t/A_t^{1/(1-\theta)}$, etc. Denote the steady state values of the detrended variables by an upper bar.

The dynamics (69) is then rewritten as:

$$\left(\frac{\gamma}{\beta}\left(\frac{\hat{K}_{t+1}}{\hat{K}_t}\right)^{\theta}-1+\delta\right)\hat{K}_{t+1}=\frac{a^{1-\rho}}{\eta a_y}\left(b_w a_w\hat{K}_{t+1}^{\theta}+c_X(\gamma\hat{K}_{t+2}-(1-\delta)\hat{K}_{t+1})+\chi\tau\bar{Y}\right) \tag{70}$$

We further assume that agents form their expectations by log-linearly approximating the dynamics (70). We denote the deviation of the log detrended variables from the steady state values by a tilde, such as $\tilde{K}_t=\log\hat{K}_t-\log\bar{K}$. By a first-order log-linear approximation, (70) becomes:

$$\frac{\bar{R}\theta(\tilde{K}_{t+1}-\tilde{K}_t)}{\bar{R}-1+\delta}+\left(\bar{R}-1+\delta\right)\tilde{K}_{t+1}=\frac{a^{1-\rho}}{\eta a_y}\left(b_w\left(\frac{\bar{w}}{\bar{K}}\right)\theta\tilde{K}_{t+1}+c_X\left(\gamma\tilde{K}_{t+2}-(1-\delta)\tilde{K}_{t+1}\right)\right) \tag{71}$$

Rearranging, we obtain:

$$\begin{bmatrix}\tilde{K}''\\ \tilde{K}'\end{bmatrix}=\begin{bmatrix}\left(\frac{\eta a_y}{a^{1-\rho}c_X\gamma}\right)\left(\frac{\bar{R}\theta}{\bar{R}-1+\delta}+\bar{R}-1+\delta\right)-\left(\frac{b_w\theta}{c_X\gamma}\frac{\bar{w}}{\bar{K}}-\frac{1-\delta}{\gamma}\right) & -\frac{\bar{R}\theta\eta a_y}{(\bar{R}-1+\delta)a^{1-\rho}c_X\gamma}\\ 1 & 0\end{bmatrix}\begin{bmatrix}\tilde{K}'\\ \tilde{K}\end{bmatrix} \tag{72}$$

Numerical computations show that the coefficient matrix has one eigenvalue greater than 1 and the other less than 1 for the parameter sets we consider. Thus the dynamics is determinate. We pick the smaller root $\eta_K<1$ to form the log-linearized expected dynamics $\tilde{K}' = \eta_K \tilde{K}$.

The approximated equilibrium path solves the system (38–46) where the perfect foresight condition (40) is replaced with the approximated forecast $\tilde{K}^e_{t+1} = \eta_K \tilde{K}_t$. The forecasted capital sequence $(\tilde{K}_t, \tilde{K}^e_{t+1})$ determines the prices w_t and R_{t+1}, and then the rest of the system is solved.

Due to the high-dimensional nonlinearity of the capital profile dynamics, there may be multiple profiles of $(k_{j,t+1})$ that solve the system given the profile $(k_{j,t})$. We thus need to specify an equilibrium selection algorithm. We select an equilibrium path that is the closest to the equilibrium path in the economy with a continuum of firms. Namely, we select $\tilde{K}_{t+1}$ that is the closest to the expected aggregate capital $\eta_K \tilde{K}_t$ among all the $\tilde{K}_{t+1}$'s that solve the equation system given $(k_{j,t})$.

This selection ensures that our estimate of the magnitude of endogenous fluctuations is most conservative, since we choose the least volatile equilibrium path. In particular, this selection excludes the fluctuations that arise from some informational coordination among firms. The selected equilibrium can be reached by a fictitious best response dynamics in which firms only need to know the aggregate capital to make their decisions. Thus, the informational requirement for firms' decisions is quite parsimonious. We argue, along with Cooper (1994), that such parsimony is a desirable property for an equilibrium selection algorithm to have in macroeconomic analysis, because it would cost a lot for firms to collect precise information on the capital profile, and because it would require an extensive communication for the many, heterogeneous firms to coordinate their expectations.

3 Numerical Results

3.1 Calibration

We use steady-state values to calibrate fundamental parameters. Those "great ratios" are obtained as follows. From (66), we obtain a steady-state output-capital ratio as:

$$a_{YK} \equiv \frac{\bar{Y}}{\bar{K}} = \eta a^{\rho-1}\left(\frac{\gamma}{\beta} - 1 + \delta\right). \tag{73}$$

The dynamics (70) has a steady state:

$$\bar{K} = \left(\frac{b_w a_w}{(a_y - \chi\tau)a_{YK} - c_X(\gamma - 1 + \delta)}\right)^{1/(1-\theta)}. \tag{74}$$

The steady state wage is given by $\bar{w} = a_w \bar{K}^{\theta}$. Then,

$$\frac{\bar{w}}{\bar{K}} = \frac{(a_y - \chi\tau)a_{YK} - c_X(\gamma - 1 + \delta)}{b_w}. \tag{75}$$

By using (73) and the relation $\bar{X}/\bar{K} = c_X(\gamma - 1 + \delta)$ from (64), we obtain:

$$a_{XY} \equiv \frac{\bar{X}}{\bar{Y}} = \frac{a^{1-\rho}c_X(\gamma - 1 + \delta)}{\eta(\gamma/\beta - 1 + \delta)}. \tag{76}$$

Using the steady-steate relation of (19), we obtain the steady-state share of labor income:

$$\bar{w}\bar{H}/\bar{Y} = 1 - 1/\eta + \underline{h}\bar{w}/\bar{Y} \tag{77}$$

$$= 1 - 1/\eta + \underline{h}\frac{(a_y - \chi\tau)a_{YK} - c_X(\gamma - 1 + \delta)}{b_w \eta a^{\rho-1}(\gamma/\beta - 1 + \delta)}. \tag{78}$$

To calibrate β, we use the steady-state real interest rate $\bar{R} = \gamma/\beta$ from (31). $\bar{R}$ is set at 1.6% and the trend growth rate γ is set at 0.37% quarterly.

We match the consumption-output ratio and the investment-output ratio by calibrating θ. We set $\bar{C}/\bar{Y} = 0.59$ and $\bar{X}/\bar{Y} = 0.18$. In the benchmark model (Specifications I and II below), we abstract from the government sector, and thus we set $\tau = 0$ and a_{XY} to be $\bar{X}/(\bar{C}+\bar{X}) = 0.18/(0.59+0.18)$. The government is incorporated in Specifications III and IV, where we set $\tau = 1 - 0.59 - 0.18 = 0.23$.

Labor income share is set at 0.58 and matched by calibrating $\underline{h}$. Since aggregate profits (dividend payments to households) satisfy $\Pi_t = C_t - w_t H_t$, the profit share of output is equal to the consumption share less labor income share. Under the calibration $\bar{C}/\bar{Y} = 0.59$ and $\bar{w}\bar{H}/\bar{Y} = 0.58$, the profit share is 1%.

The cross-section distribution of the lumpiness parameter λ_i is matched with the empirical observations reported in Cooper, Haltiwanger, and Power (1999). The distribution shape is fitted well by an exponential distribution. Following Cooper et al., we regard $\lambda_i = 1.2$ as the characteristic size of lumpiness. Thus we set the mean of the distribution at 1.2, its standard deviation at 0.1, and its lower bound at 1.1. N is set at 350000, which is the number of manufacturing establishments in the US (Cooper, Haltiwanger, and Power (1999)). The capital depreciation rate is set at the standard 0.025 quarterly. The fraction of rational households, χ, is set at 50% in Specifications I and III, and at 10% in II and IV. The parameter for the elasticity of substitution η is freely chosen to adjust the variance of aggregate investment. We set $\eta = 1.57$ for I and II and $\eta = 1.38$ for III and IV.

3.2 Simulations

The equilibrium path is simulated for 300 quarters and the first 100 quarters are discarded. The equilibrium path is detrended by the Hodrick-Prescott filter at the smoothing parameter 1600, and then equilibrium moments are computed. Each run is repeated for 30 times to obtain average and standard deviation of the equilibrium

$\bar{w}\bar{H}/\bar{Y}$	$\bar{R}-1$	γ	β	δ	mean λ	std λ
0.58	0.016	1.0037	0.9878	0.025	1.2	0.1

Table 1: Calibrated moments

	χ	τ	η	a_{XY}	θ	$\underline{h}$
I	0.5	0	1.57	0.2338	0.9225	0.2278
II	0.1	0	1.57	0.2338	0.9225	0.0888
III	0.5	0.23	1.38	0.18	0.9365	0.3670
IV	0.1	0.23	1.38	0.18	0.9365	0.1108

Table 2: Parameter values

moments.

Table 3 summarizes the moment properties of the equilibrium paths. We observe that endogenous fluctuations of investment, consumption, output, and capacity utilization (hours) occur. The standard deviation of output relative to investment also roughly matches with the empirical counterpart. Consumption and investment are strongly procyclical. By comparing across different specifications, we observe that a larger fraction of rule-of-thumb households (smaller χ) leads to a larger standard deviation of output, and an increase in η reduces the fluctuation magnitudes. Fluctuations of capital and wage are as small as those in the business cycles, while the magnitude of consumption fluctuations is considerably smaller than data. We observe that the model generates no autocorrelations.

4 Mechanism of endogenous fluctuations

In this section, we analytically characterize the mechanism of the endogenous fluctuations we observed in the numerical simulations. The analysis here is similar to the formal analysis of different models presented in separate papers (Nirei (2006), Nirei

	$\hat{Y}$	$\hat{C}$	$\hat{X}$	$\hat{H}$	$\hat{K}$	$\hat{W}$
I	($\tau = 0$, $\eta = 1.57$, $\chi = 0.5$)					
Std. dev.	0.0107	0.0023	0.0386	0.0070	0.0011	0.0010
(se)	(0.0005)	(0.0001)	(0.0018)	(0.0003)	(0.0001)	(0.0001)
Corr. with $\hat{Y}$	1	0.9534	0.9981	0.9967	-0.4390	-0.4390
(se)		(0.0055)	(0.0003)	(0.0004)	(0.0328)	(0.0328)
Autocorr.	-0.0617	0.0385	-0.0664	-0.0667	0.4727	0.4727
(se)	(0.0717)	(0.0722)	(0.0716)	(0.0715)	(0.0508)	(0.0508)
II	($\tau = 0$, $\eta = 1.57$, $\chi = 0.1$)					
Std. dev.	0.0147	0.0060	0.0433	0.0095	0.0012	0.0011
(se)	(0.0007)	(0.0003)	(0.0022)	(0.0005)	(0.0001)	(0.0001)
Corr. with $\hat{Y}$	1	0.9955	0.9989	0.9979	-0.4640	-0.4640
(se)		(0.0008)	(0.0002)	(0.0004)	(0.0389)	(0.0389)
Autocorr.	-0.0866	-0.0706	-0.0898	-0.0897	0.4556	0.4556
(se)	(0.0722)	(0.0736)	(0.0721)	(0.0719)	(0.0676)	(0.0676)
III	($\tau = 0.23$, $\eta = 1.38$, $\chi = 0.5$)					
Std. dev.	0.0034	0.0007	0.0169	0.0017	0.0005	0.0005
(se)	(0.0002)	(0.0000)	(0.0008)	(0.0001)	(0.0000)	(0.0000)
Corr. with $\hat{Y}$	1	0.8410	0.9971	0.9936	-0.4259	-0.4259
(se)		(0.0105)	(0.0003)	(0.0006)	(0.0246)	(0.0246)
Autocorr.	-0.0693	0.2109	-0.0757	-0.0762	0.4750	0.4750
(se)	(0.0589)	(0.0510)	(0.0592)	(0.0593)	(0.0396)	(0.0396)
IV	($\tau = 0.23$, $\eta = 1.38$, $\chi = 0.1$)					
Std. dev.	0.0090	0.0035	0.0389	0.0045	0.0011	0.0010
(se)	(0.0005)	(0.0002)	(0.0024)	(0.0003)	(0.0001)	(0.0001)
Corr. with $\hat{Y}$	1	0.9686	0.9971	0.9952	-0.4339	-0.4339
(se)		(0.0041)	(0.0004)	(0.0007)	(0.0367)	(0.0367)
Autocorr.	-0.0782	0.0010	-0.0850	-0.0851	0.4599	0.4599
(se)	(0.0805)	(0.0824)	(0.0804)	(0.0803)	(0.0602)	(0.0602)

Table 3: Simulated moments

(2008)). To facilitate the analysis, we introduce a fictitious tatonnement process that starts from the capital profile $(k_{j,t})$ and results in $(k_{j,t+1})$ as follows.

1. Initialize step $v = 0$ and $k_{j,t+1,0} = k_{j,t}$.
2. Given $\hat{K}_t$, firms predict the next period aggregate capital $K_{t+1,0}$ as $\hat{K}_{t+1,0} = \eta_K \hat{K}_t$.
3. Form the adjustment threshold based on the predicted aggregate capital: $k^*_{j,t+1,v} = (a_j/a)K_{t+1,v}$.
4. Adjust capital according to the threshold rule:

$$k_{j,t+1,v+1} = \begin{cases} \lambda_j(1-\delta)k_{j,t+1,v} & \text{if } (1-\delta)k_{j,t+1,v} \leq k^*_{j,t+1,v}, \\ (1-\delta)k_{j,t+1,v} & \text{if } k^*_{j,t+1,v} < (1-\delta)k_{j,t+1,v} \leq \lambda_j k^*_{j,t+1,v}, \\ (1-\delta)k_{j,t+1,v}/\lambda_j & \text{if } (1-\delta)k_{j,t+1,v} > \lambda_j k^*_{j,t+1,v}. \end{cases} \quad (79)$$

5. Stop the procedure if $v > 0$ and if there were no firms that adjusted capital in 4. Equilibrium outcome is $K_{t+1} = K_{t+1,v}$ and $k_{j,t+1} = k_{j,t+1,v}$. Otherwise, set the step forward to $v + 1$, update aggregate capital as $K_{t+1,v+1} = (\sum_{j=1}^{N} k^{\rho}_{j,t+1,v+1}/N)^{1/\rho}$, and repeat from 3.

We can show that the converged capital profile of the tatonnement process above coincides with an equilibrium profile which is selected by an alternative equilibrium selection mechanism. The alternative equilibrium selection chooses $\tilde{K}_{t+1}$ closest to $\eta_K \tilde{K}_t$ such that $\text{sign}(\hat{K}_{t+1} - \eta_K \hat{K}_t) = \text{sign}(\hat{K}_{t+1,0} - \eta_K \hat{K}_t)$. That is, it selects the equilibrium capital closest to the expected capital in the direction of the initial expectation error.

The fluctuation observed under the alternative equilibrium selection is similar to that observed previously. Table 4 shows the simulated moments under the alternative equilibrium selection. We observe that the fluctuations are larger than the previous simulations, naturally. Other than that, the fluctuation pattern is quite similar to the previous one. The alternative selection is closely connected to the original selection, as the equilibrium selected in the previous section can be achieved by running the new selection algorithm for the both directions below and above $\eta_K \tilde{K}_t$. In what follows, we utilize the alternative equilibrium selection in order to characterize the endogenous fluctuations analytically.

The total capital growth in a period is determined by two components in the fictitious tatonnement process: the initial adjustments caused by the initially expected growth $\log K^e_{t+1} - \log K_t$, and the successive adjustments caused by the initial responses. In the initial step, capitals are depreciated by δ, firms form expectations for the aggregate capital next period K^e_{t+1}, and a fraction of them decide to undertake lumpy investments. The actual aggregate capital after the initial investment would be equal to the expected capital if there were a continuum of firms distributed uniformly over the inaction band. Since there are only a finite number of firms, however, there will be a slight difference between the actual and the expected. The gap will be filled by the successive adjustments in the tatonnement.

We characterize the initial and successive contributions separately and then assess the total impact. First, we analytically characterize the number of firms that invest in the initial step, which we call m_1. To facilitate the characterization, we regard the actual configuration of firms' positions in the inaction band as N random draws from the uniform distribution over the inaction band. Firm i then invests with probability $1/q_i$ at the stationary level of capital $\hat{K}_t = \bar{K}$. Then m_1 follows the summation of the Bernoulli trials with probability $1/q_i$ over i. The variance of m_1 is $\sum_{i=1}^{N}(1 - 1/q_i)/q_i$, and the variance of m_1/N decreases linearly in N.

The size of fluctuation of the successive adjustments is determined by the sensitivity of k^*, the lower bound of the inaction band, to the aggregate capital K. From (18), we see that there are two channels of the response: $K^{-\rho}$ and Y. The first is the price effect: an increase in capital raises the real wage and thus reduces the labor demand and the

	$\hat{Y}$	$\hat{C}$	$\hat{X}$	$\hat{H}$	$\hat{K}$	$\hat{W}$
I	($\tau = 0$, $\eta = 1.57$, $\chi = 0.5$)					
Std. dev.	0.0120	0.0026	0.0433	0.0079	0.0013	0.0012
(se)	(0.0006)	(0.0001)	(0.0021)	(0.0004)	(0.0001)	(0.0001)
Corr. with $\hat{Y}$	1	0.9518	0.9980	0.9966	-0.4530	-0.4530
(se)		(0.0047)	(0.0002)	(0.0004)	(0.0293)	(0.0293)
Autocorr.	-0.0167	0.0756	-0.0216	-0.0203	0.4792	0.4792
(se)	(0.0633)	(0.0638)	(0.0629)	(0.0631)	(0.0468)	(0.0468)
II	($\tau = 0$, $\eta = 1.57$, $\chi = 0.1$)					
Std. dev.	0.0150	0.0061	0.0443	0.0097	0.0013	0.0012
(se)	(0.0009)	(0.0004)	(0.0026)	(0.0006)	(0.0001)	(0.0001)
Corr. with $\hat{Y}$	1	0.9948	0.9986	0.9976	-0.4209	-0.4209
(se)		(0.0007)	(0.0002)	(0.0003)	(0.0366)	(0.0366)
Autocorr.	-0.1052	-0.0833	-0.1106	-0.1102	0.5062	0.5062
(se)	(0.0795)	(0.0805)	(0.0792)	(0.0793)	(0.0604)	(0.0604)
III	($\tau = 0.23$, $\eta = 1.38$, $\chi = 0.5$)					
Std. dev.	0.0034	0.0007	0.0170	0.0017	0.0005	0.0005
(se)	(0.0002)	(0.0000)	(0.0011)	(0.0001)	(0.0000)	(0.0000)
Corr. with $\hat{Y}$	1	0.8366	0.9970	0.9935	-0.4267	-0.4267
(se)		(0.0161)	(0.0004)	(0.0009)	(0.0328)	(0.0328)
Autocorr.	-0.0106	0.2394	-0.0147	-0.0153	0.4841	0.4841
(se)	(0.0665)	(0.0715)	(0.0664)	(0.0663)	(0.0546)	(0.0546)
IV	($\tau = 0.23$, $\eta = 1.38$, $\chi = 0.1$)					
Std. dev.	0.0082	0.0031	0.0357	0.0041	0.0009	0.0008
(se)	(0.0004)	(0.0002)	(0.0019)	(0.0002)	(0.0001)	(0.0001)
Corr. with $\hat{Y}$	1	0.9775	0.9981	0.9967	-0.5267	-0.5267
(se)		(0.0037)	(0.0004)	(0.0006)	(0.0342)	(0.0342)
Autocorr.	-0.2284	-0.1695	-0.2326	-0.2323	0.3026	0.3026
(se)	(0.0615)	(0.0645)	(0.0615)	(0.0611)	(0.0531)	(0.0531)

Table 4: Simulated moments under the alternative equilibrium selection

optimal supply. The second is the income effect: an increase in aggregate demand increases the optimal goods supply. Note that another channel from real interest rate is shut off by the assumption of the predetermined interest: the discount rate applied by firms has to be decided before the firms decide investments. By log-linearizing the aggregate demand function $a_y Y_t = b_w a_w A_t K_t^\theta + X_t + G_t$ and combining with (64), we obtain:

$$\eta_{KY} \equiv \frac{d \log Y_t}{d \log K_t} = \frac{1}{a_y} \left(b_w \theta \frac{\bar{w}}{\bar{Y}} + c_X (\gamma \eta_K - 1 + \delta) \frac{\bar{K}}{\bar{Y}} \right) \tag{80}$$

where the first term represents the effect through an increase in wage and the second term shows the effect through future investment.

Then, $\eta_{Kk} \equiv d \log k_t^* / d \log K_t = (\eta_{KY} - \rho)/(1 - \rho)$. Thus, the probability that firm i is induced to invest by a one-percent increase in aggregate capital is $\eta_{Kk} / \log \lambda_i$. Also, an increase of $\log K_t$ by j's investment is approximately equal to $\log \lambda_j / N$ when N is large. In a situation where the heterogeneity in λ_j is negligible, then, the number of firms that are induced to invest by a single firm's investment follows a binomial distribution with probability parameter η_{Kk}/N and population N. Thus, the alternative equilibrium selection algorithm can be embedded in a branching process with the binomial. Namely, the number of firms who invest in step $u + 1$ is the binomial distribution convoluted by the number of firms who invest in u. The branching process will stop in a finite step with probability 1 if the mean of the binomial is less than or equal to 1 (see Feller (1957)). Thus, for the equilibrium selection to be valid, we need $0 < \eta_{Kk} \leq 1$.

Suppose that $0 < \eta_{Kk} \leq 1$ is satisfied, and take N to infinity. Then, the equilibrium selection algorithm asymptotically follows a Poisson branching process: the number of firms induced to invest by a single firm in each step follows a Poisson distribution with mean η_{Kk}. Let W denote the total number of firms that are induced to invest in the entire process that starts from one firm. Let F denote the probability generating function of W, and G denote the probability generating function of the Poisson distribution. Then, for the branching process, a recursive relation $F(s) = sG(F(s))$ holds. Thus, $E(W) = F'(1) = 1/(1-\eta_{Kk})$ and $E(W(W-1)) = F''(1) = \eta_{Kk}(2-\eta_{Kk})/(1-\eta_{Kk})^3$. The total number of investing firms is m_1 plus m_1-convolution of W which we write as W^{*m_1}. We obtain:

$$V(m_1 + W^{*m_1}) = E(V(m_1 + W^{*m_1} \mid m_1)) + V(E(m_1 + W^{*m_1} \mid m_1)) \tag{81}$$

$$= E(|m_1|)V(W) + (1 + E(W))^2 V(m_1) \tag{82}$$

$$= \sqrt{2V(m_1)/\pi}\eta_{Kk}/(1-\eta_{Kk})^3 + ((2-\eta_{Kk})/(1-\eta_{Kk}))^2 V(m_1) \tag{83}$$

Suppose that the lumpiness λ_j is homogenous across firms. Then $V(m_1) = N(1 - 1/q)/q$. A normalized capital growth rate, $N(\hat{K}_{t+1} - \hat{K}_t)$, is asymptotically equal to the lumpiness $\log \lambda$ multiplied by the fraction of firms that invest. Then the variance of the growth rate $V(\hat{K}_{t+1} - \hat{K}_t)$ becomes:

$$(\log \lambda)^2 \left(\sqrt{2(1-1/q)/q\pi}\eta_{Kk}/(1-\eta_{Kk})^3 N^{1.5} + ((2-\eta_{Kk})/(1-\eta_{Kk}))^2 (1-1/q)/qN \right) \tag{84}$$

As N tends to infinity, the second term dominates the first term. The second term declines linearly in N, and thus the law of large number is holding: the aggregate variance decreases linearly in N.[1] When N is finite, however, the variance can be non-negligible if η_{Kk} is close to one, because the second term has a square of the inverse of $1 - \eta_{Kk}$ and the first term has its cube. Thus, a considerable aggregate fluctuation may arise for a large N if the strategic complementarity η_{Kk} is close enough to one.

This result is an extension of the research started by Jovanovic (1987) who formulates the idea that idiosyncratic shocks can generate aggregate shocks by a strong

[1] We can show that the law of large numbers does not hold if $\eta_{Kk} = 1$. Namely, the variance of the aggregate variables does not converge to zero as N tends to infinity. This case is investigated in a separate paper Nirei (2008).

multiplier effect. Our result differs from Jovanovic's in that the multiplier effect is not constant but depends sensitively on the detailed configuration the capital profile. Thus, the aggregate fluctuations occur endogenously in our model, while Jovanovic's model does not generate fluctuations if there were no exogenous shocks. Our result is closely connected to Durlauf (1993) who emphasized the effects of detailed states in high-dimensional nonlinear dynamics.

Note that in (84) the lumpiness $\log \lambda$ affects the standard deviation of the aggregate growth rate almost linearly ($\log \lambda$ also affects q but its effect is quantitatively negligible). The aggregate fluctuation is further enhanced by the dispersion of λ_j when λ_j is heterogeneous. In that case, the recursive relation of the generating function $F(s) = sG(F(s))$ is reinterpreted as G being a generating function of a compound Poisson distribution. The computation of the variance is possible but tedious (Nirei (2003)).

Lumpiness is quantitatively important to generate the aggregate fluctuations. We note that the magnitude of the micro-level fluctuation caused by the lumpiness is independent of time scale, whereas idiosyncratic quarterly shocks in productivity will be smaller than annual shocks. Thus, it is natural to guess that the lumpiness affects the short-run fluctuations whereas the productivity shocks take over in the long run. It is possible to incorporate the idiosyncratic productivity shocks in our model and generate the aggregate fluctuations through the same propagation mechanism. Adding the idiosyncratic shocks would generate even larger aggregate fluctuations, but the lumpiness will continue to provide an important source of fluctuations in the short run.

The endogenous fluctuation of capital affects output through investment demand. Let $\sigma(g_K) = \sqrt{V(\hat{K}_{t+1} - \hat{K}_t)}$ denote the standard deviation of the capital growth fluctuation. In the benchmark specification I, the analytic approximation above gives $\sigma(g_K) = 0.0013$. This could generate a large enough fluctuation of investment and output, because Y and X are substantially smaller than K in quarterly basis. Note that $\hat{X} \simeq (X-\bar{X})/\bar{X} = (g_K K+\delta(K-\bar{K}))/\delta\bar{K}$. Around the steady state $K = \bar{K}$, $\sigma(\hat{X})$ is roughly about $\sigma(g_K)/\delta$, which is equal to 0.052 under the calibrated parameters. This generates the fluctuation of output $\sigma(\hat{Y})$ to be about $0.052\bar{X}/\bar{Y} = 0.012$, provided that the consumption is fixed. If the consumption comoves with the investment demand, then the output fluctuation will be even larger.

Given the autonomous investment fluctuations, the relative size of fluctuations of Y and C are determined from the relations: $a_y Y_t = b_w w_t + X_t + G_t$ and $C_t = b_w w_t + (1-a_y)Y_t - (1-\chi)T_t$. In terms of log-deviation, we have: $a_y\tilde{Y}_t = b_w(\bar{w}/\bar{Y})\tilde{w}_t + (\bar{X}/\bar{Y})\tilde{X}_t$. Note that the fluctuation of wage solely arises from capital movements. Thus, wage and investment have little correlation around the stationary-level capital. If we neglect the correlation term, then we have $V(\tilde{Y})/V(\tilde{X})$ approximately equal to $(b_w a_{wY}/a_y)^2 V(\tilde{w})/V(\tilde{X}) + (a_{XY}/a_y)^2$, Also by $a_{CY}\tilde{C}_t = b_w a_{wY}\tilde{w}_t + (1-a_y)\tilde{Y}_t$, we can approximate $V(\tilde{C})/V(\tilde{Y})$ by $((1-a_y)/a_{CY})^2 + (b_w a_{wY})^2 V(\tilde{w})/V(\tilde{Y})$. The relative variances are observed roughly at $V(Y)/V(X) = 1/9$ and $V(C)/V(Y) = 0.6$ in the business cycles. These values can be matched if we set $a_y \approx 0.5$, which requires $\eta > 2$ and to set χ accordingly. In our current calibration, however, we obtain small fluctuations of investment when η is higher than 2. For the range of η that generates considerable fluctuations, we obtain reasonable simulated results for $V(Y)/V(X)$, but a very small number for $V(C)/V(Y)$.

Simulations show that the model requires a high mark-up rate and a high capital intensiveness in order to generate the large fluctuations. This is due to the fact that the amplification mechanism relies heavily on the persistence of aggregate capital. When the aggregate capital is persistent, an increase in investment this period raises the investment in the next period, which increases the output in the next period and thus strengthens the incentive to invest in this period. To achieve high η_K, a high value of θ is required. In order to maintain the labor share under high θ, a high value of η is required. This problem might be alleviated by incorporating other effects of investment on output. For example, investment may affect next period's consumption through increased employments. Such an extension is left for future research.

5 Conclusion

This paper presents a real dynamic general equilibrium model with monopolistic competition, variable capacity utilization, and lumpy investments. The model does not have any exogenous shocks, and yet it is able to generate the aggregate fluctuations due to the non-linearity that arises from the lumpy investments. In this sense, this paper provides a theory of endogenous fluctuations of the Solow residuals that are taken exogenous in the standard literature.

Under the standard calibration of first moments of aggregate variables, the model generates aggregate fluctuations of investment and output that are comparable in magnitude to the fluctuations observed at the business cycles frequencies. The comovement of output, investment, and consumption is also captured well. However, the magnitude of consumption fluctuation is smaller than its empirical counterpart. Also, the model generates no autocorrelations. The calibration used is not quite comparable with empirical observations on the mark-up rate and capital intensiveness. A possible extension of the model is suggested to address these problems.

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